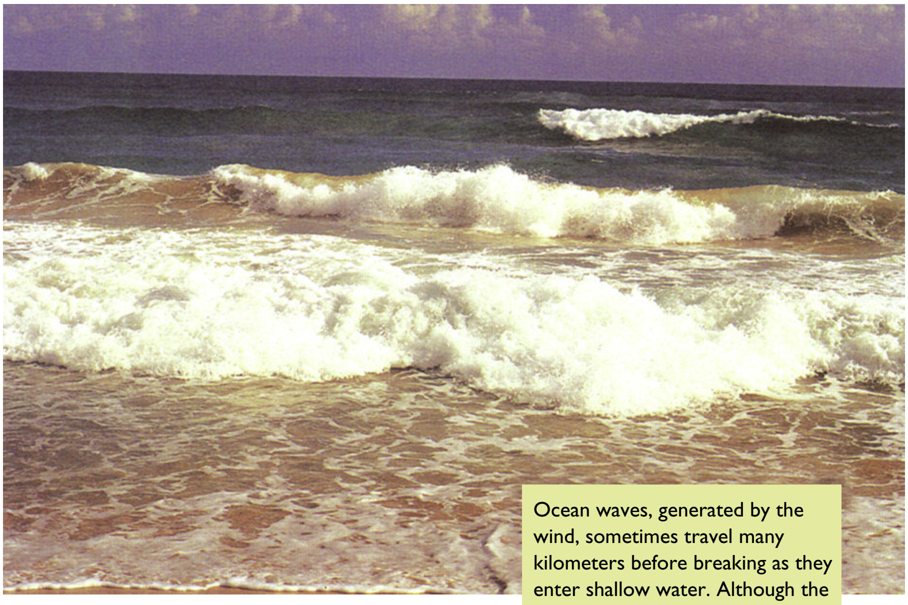
CHAPTER 16 Mechanical Waves; Sound



Ocean waves, generated by the wind, sometimes travel many kilometers before breaking as they enter shallow water. Although the wave and its energy move over great distances, the water experiences only small-amplitude, periodic motion.

The world around us is filled with waves—sound waves, radio waves, microwaves, X-rays, light waves, water waves, earthquake waves, and many others. Some of these waves require a material medium for their transmission. These are called **mechanical waves.** Water waves, sound, and earthquake waves are all mechanical waves, since each requires a medium through which to propagate. Water waves travel through water, and earthquake waves travel through the earth. Sound travels through air or some other medium. None of these waves can be transmitted through a vacuum.

In this chapter we shall study only mechanical waves, in particular, water waves, sound, and waves on a string. In later chapters we shall study radio waves, microwaves, X-rays, and visible light, all of which are examples of electromagnetic waves. Electromagnetic waves are not mechanical waves. No physical medium is necessary to transmit these waves; they can travel through a vacuum. For example, when we look at a starry sky, we see light that has traveled through the vacuum of interplanetary space.

Although we consider only mechanical waves in this chapter, we shall find that many of the wave concepts learned here are applicable to electromagnetic waves as well. For example, all kinds of waves can transmit energy.

16-1 Description of Waves

Wave Pulses; Dominoes, Strings, and Springs

The phenomenon of a **wave pulse** is easily demonstrated with a line of dominoes, standing on end and closely spaced (Fig. 16-1). If the domino at the left end is pushed over to the right, the effect is transmitted down the line. All the dominoes fall in turn.

Each domino has kinetic energy only for the brief interval during which it is falling. A pulse of energy is transmitted from one end of the line to the other, though each domino moves only a very short distance. This is a general characteristic of mechanical wave motion: **energy is transported through matter without the transport of the matter itself.** A mechanical wave transmits energy from one place to another while the matter through which it is transmitted remains in place.

Fig. 16-1 When the domino at the left end of this line is pushed over to the right, a wave pulse passes through the line.

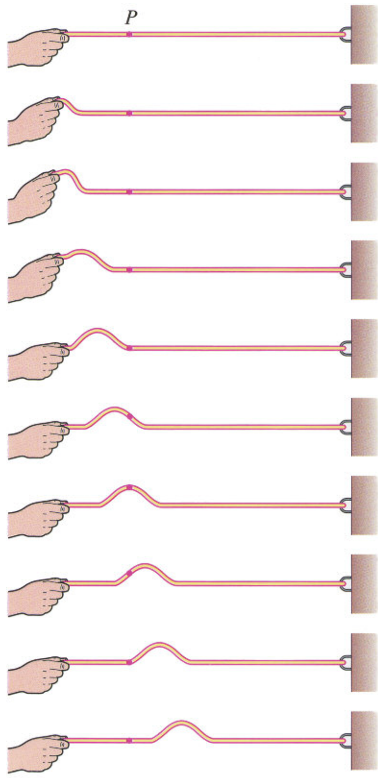


Fig. 16-2 A quick up-and-down motion of the left end of this string produces a wave pulse that travels horizontally along the string to the right. As the wave pulse passes each point on the string, that point undergoes the same up-and-down motion as the left end.

A wave pulse can also be generated on a string under tension. If the left end of a taut string is moved up and down once with a quick flip of the wrist, a wave pulse of fixed shape moves to the right at constant velocity (Fig. 16-2). *This motion of the wave pulse is not the same as the motion of the string.* As illustrated in the figure, a point P on the string moves vertically while the wave moves horizontally and transmits energy along the string. A wave such as this, in which the motion of the medium is perpendicular to the wave motion, is called a **transverse wave.**

Waves for which the motion of the medium is parallel to the direction of wave propagation are called **longitudinal waves.** Longitudinal waves on a spring are illustrated in Fig. 16-3. If the left end of the stretched, horizontal spring is pushed to the right and then pulled back to the left, the adjacent section of the spring first compresses and then stretches. This motion is transmitted to the right all along the spring: as the wave pulse passes, each section moves first to the right and then to the left. This passage of the pulse results in first a compression and then a stretching, or "rarefaction," of the spring.

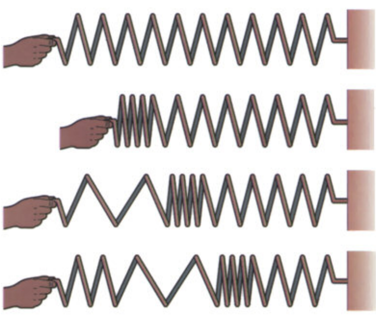


Fig. 16-3 A quick back-and-forth horizontal motion of the left end of this spring produces a wave pulse that travels along the spring to the right. As the wave pulse passes each segment of the spring, that segment undergoes the same back-and-forth motion as the left end.

Water Waves: Two-Dimensional Waves

Fig. 16-4 A wave produced by a raindrop falling on the still surface of a lake.

The preceding examples were one dimensional waves; that is, the wave energy moved along a line. A water wave is an example of wave motion in two dimensions. When the surface of a body of water is disturbed, a wave propagates radially outward (in two dimensions) from the disturbance along the surface of the water. For example, a water wave is produced by a raindrop striking the surface of a lake, as shown in Figs. 16-4 and 16-5. As the wave moves outward from the point of disturbance, the energy being transmitted by the wave spreads out and the height of the wave gradually diminishes. This is true for all two-dimensional waves as well as for all three-dimensional waves, which we consider next.

Sound Waves: Three-Dimensional Waves

A sound wave in air is an example of a three-dimensional, longitudinal wave. It consists of compressions and rarefactions of air molecules spreading out in all directions. The click you hear when two billiard balls collide is a sound-wave pulse. Air molecules are compressed between the colliding balls. This compression is followed by a rarefaction, a volume in which the density of the air molecules is much lower than in undisturbed air. The effect is transmitted outward to the surrounding air, and the wave pulse propagates in all directions.

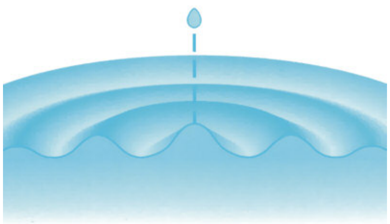


Fig. 16-5 Side view of the wave produced by a raindrop on the surface of a lake.

If a sound wave originates at a point and propagates outward equally in all directions, the wave disturbance takes the shape of a spherical surface centered on the source, as indicated in Fig. 16-6a. This is called a **spherical wave.**

If a wave disturbance is the same everywhere over the surface of a plane, the wave is said to be a **plane wave.** If a small part of a spherical wave is viewed at a large distance from the source, this part of the spherical surface is approximately a plane, and the wave may be represented by a plane wave in this region (Fig. 16-6b). A plane wave varies only in one direction—its direction of motion, which is perpendicular to the plane. Since variation in a plane wave depends on only one spatial variable, it may be described in the same way as the variation in a one-dimensional wave.

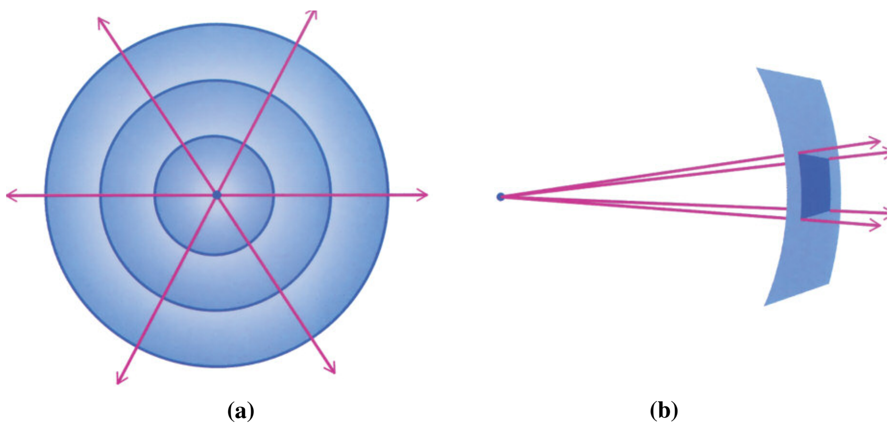


Fig. 16-6 **(a)** A sound wave originating from a point spreads out spherically. **(b)** At a great distance from the source of a spherical wave, small sections of the spherical surfaces are approximately planes.

Periodic Waves; Wavelength, Frequency, and Speed

Many waves are repetitive, or periodic. Periodic motion of the wave's source results in a wave whose form is periodic in space. For example, you can generate a periodic wave on a string by repeatedly flicking the string up and down. Some periodic wave forms are shown in Fig. 16-7. A wave form may represent the observed shape of a wave on a string, or it may represent the spatial variation of any other kind of wave disturbance. For example, the wave form may represent the spatial variation in the density of air molecules in a sound wave.

The **wavelength** of a wave, denoted by the Greek letter λ (lambda), is the distance between any two successive identical points on the wave—from one crest to the next, say, or from one trough to the next (Fig. 16-7).

If the motion of the source is SHM, the wave form that results has a sine wave shape (Fig. 16-7a) and is called a **harmonic wave.** A harmonic wave on a string is generated when one end of the string is repeatedly moved up and down in SHM (Fig. 16-8a). A vibrating tuning fork generates a harmonic sound wave (Fig. 16-8b). Harmonic waves are of particular importance in the study of waves, as we shall see in Section 16-6 when we study superposition of waves.

As a periodic wave passes a given particle in the medium, that particle undergoes periodic motion. The **frequency** f of a periodic wave is the frequency of the periodic motion experienced by each particle of the medium. For example, each air molecule in the path of a 512 Hz sound wave vibrates at a frequency of 512 Hz.

A wave imparts the motion of the source to each particle of the medium, but for each particle this motion is delayed by the time interval required for the wave to travel to that particle. For example, as a harmonic wave travels along a string, all particles of the string undergo SHM. In Fig. 16-8a particles P and Q undergo the same SHM, but Q is $\frac{1}{2}$ cycle behind P because a time interval of $\frac{1}{2}$ period is required for the wave to travel from P to Q.

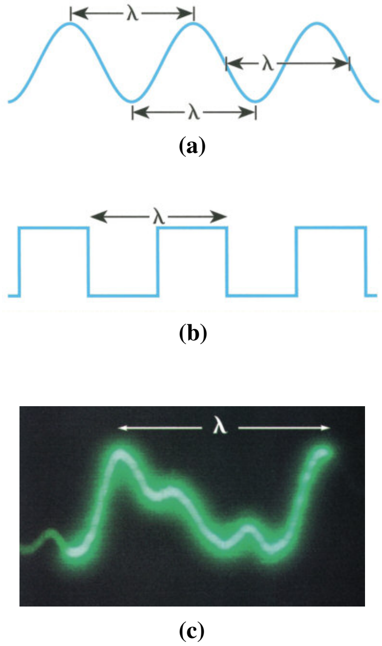


Fig. 16-7 Periodic waves: **(a)** harmonic wave; **(b)** square wave; **(c)** wave generated by a vibrating guitar string.

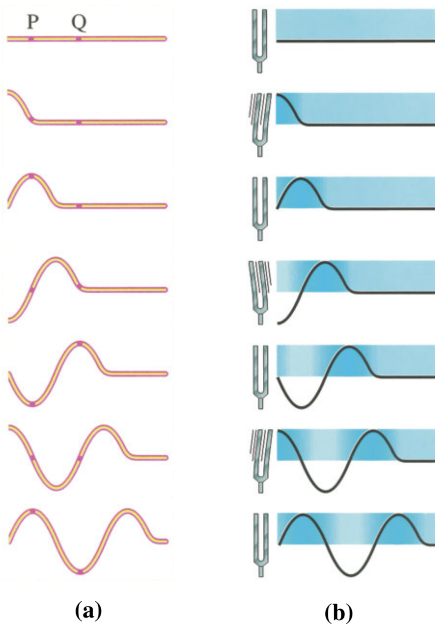


Fig. 16-8 SHM of **(a)** the end of a stretched string or **(b)** a tuning fork generates a harmonic wave.

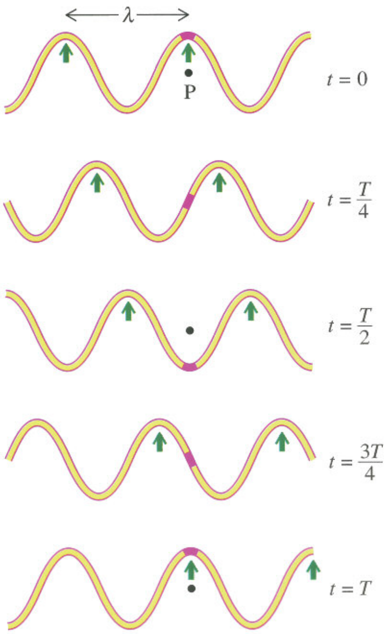


Fig. 16-9 Each segment of a string moves up and down with a period T as a wave of wavelength λ moves along the string to the right. In time T one wavelength has passed a fixed point P.

Wave speed v is the speed at which a wave propagates through the medium. It is important to understand that **wave speed is not the same as the speed of a particle of the medium.** In Fig 16-9 the colored segment of string near point P is oscillating up and down at a certain speed, but this is *not* the wave speed. The rate at which the crests labeled with arrows move to the right is the wave speed.

By relating the SHM of a particle of the string to the motion of the wave, we shall show how wavelength is related to wave speed and frequency. Fig. 16-9 shows one cycle of motion for a segment of string near point P. This segment undergoes periodic motion (up and down in the y direction) with period T. During the same time T, each crest of the wave moves a distance λ to the right. The wave crest moves to the right at the wave speed v, where

$$v = \frac{\lambda}{T}$$

Indeed, each part of the wave form moves at this speed. Since the frequency f equals $1/T$, we may express the equation above in the form

$$v = \lambda f \tag{16-1}$$

This relationship is valid for waves of any kind (water waves, sound, light, and so forth), and although we have illustrated a harmonic wave in Fig. 16-9, the relationship is valid for any wave form.

The frequency of a wave is always determined solely by the wave source. Thus, once a wave is formed, its frequency doesn't change, even when the wave passes from one medium to another. In contrast, **the speed of a wave is determined by the medium through which the wave travels.** For example, the speed of sound in water (1480 m/s) is quite different from the speed of sound in air (340 m/s). Wave speed in a given medium *may* depend on the frequency of the wave; that is, waves of some frequencies may travel faster than waves of other frequencies. This phenomenon is called **dispersion.***

If both the speed and the frequency of a wave are known, we can find its wavelength by using Eq. 16-1 ($v = \lambda f$) to solve for λ. Thus **wavelength depends on both the source and the medium.**

EXAMPLE 1 Wavelength of Sound for a Musical Note

Frequencies of sound produced by a piano range from about 30 Hz for the lowest notes to about 4000 Hz for the highest notes. Find the wavelength in air of a 262 Hz sound wave produced by striking middle C on a piano. What would the wavelength of this sound be under water? The speed of sound is 340 m/s in air and 1480 m/s in water.

SOLUTION We apply Eq. 16-1:

$$v = \lambda f$$

or

$$\lambda = \frac{v}{f}$$

In air, $$\lambda = \frac{340\ \text{m/s}}{262\ \text{Hz}} = 1.30\ \text{m}$$

In water, $$\lambda = \frac{1480\ \text{m/s}}{262\ \text{Hz}} = 5.65\ \text{m}$$

Notice that the frequency remains unchanged, even when the wave leaves the air and enters water. Since wave speed increases as the wave goes from air to water, the relationship $\lambda = \frac{v}{f}$ shows that the wavelength must increase as v increases.

*Electromagnetic waves traveling through matter also exhibit dispersion. For example, red light travels through glass faster than blue light; this is responsible for the colors one sees in looking through a prism (see Section 23-2).

Reflection

Any abrupt change in the medium a wave travels through will result in the wave's being reflected. An echo is a sound wave that reflects as its medium changes from air to some solid. If you yell in a mountainous area, you can often hear your echo a few seconds later. The sound wave produced by your voice has been reflected by the surrounding mountains. Similarly, when a wave on a string reaches the end of the string, the wave is reflected. Fig. 16-10 shows the reflection of wave pulses. If the end of the string is held fixed, as in Fig. 16-10a, the wave pulse is inverted when it is reflected. If the end is free to move, as in Fig. 16-10b, the reflected wave pulse is not inverted.

Fig. 16-11 shows a wave pulse in a changing medium, going from a lighter string to a heavier one. In this case the pulse is partially transmitted into the second medium and partially reflected back into the first medium. The reflected pulse is inverted when the second medium is denser than the first one. When the first medium is the denser one, the reflected wave is not inverted.

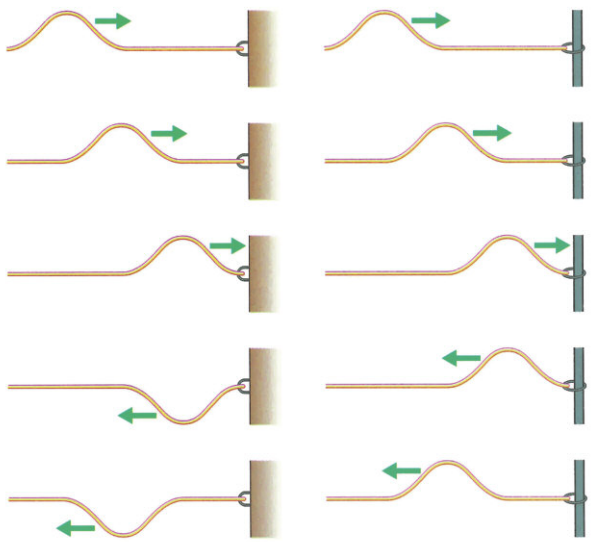


Fig. 16-10 When a wave pulse on a string reaches the string's end, the wave is **(a)** inverted if the end is fixed; **(b)** not inverted if the end is free.

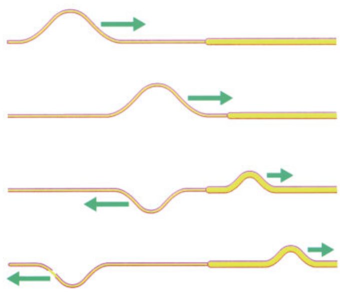


Fig. 16-11 When a wave pulse goes from one string to another of different density, the pulse is partially transmitted and partially reflected.

EXAMPLE 2 Ultrasound Imaging

Only sounds in the frequency range from about 20 Hz to about 20,000 Hz are audible to humans. Ultrasound is the name given to sound at frequencies above 20,000 Hz. Ultrasound can be used to produce images inside the human body (Fig. 16-12). Ultrasound waves penetrate the body, traveling at a speed of 1500 m/s, and are reflected from surfaces inside.

For a good ultrasonic picture having sufficient detail, the wavelength should be no greater than about 1.0 mm. Find the frequency of such an ultrasonic wave.

SOLUTION Solving Eq. 16-1 ($v = \lambda f$) for f, we obtain

$$f = \frac{v}{\lambda} = \frac{1500 \text{ m/s}}{1.0 \times 10^{-3} \text{ m}}$$

$$= 1.5 \times 10^{6} \text{ Hz} \quad (\text{or } 1.5 \text{ MHz})$$

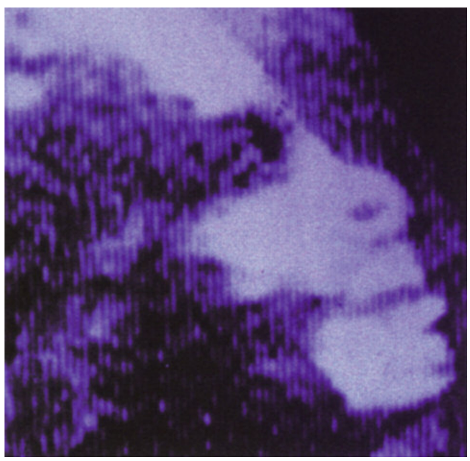


Fig. 16-12 Ultrasound image of a fetus.

16-2 Wave Speed

In this section we shall discuss the speed of propagation of waves of various types and see which characteristics of the media determine the wave speed. Again, as stressed in the preceding section, it is important to understand that what we are considering here is not the speed of individual particles of the medium but rather the speed at which the wave form moves through the medium.

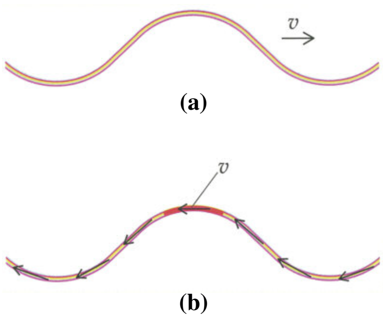


Fig. 16-13 **(a)** Viewed from the laboratory reference frame, a wave pulse moves to the right at constant speed v, while the string moves in the vertical direction as the pulse passes. **(b)** Viewed from a reference frame moving to the right at speed v with respect to the laboratory, the wave pulse appears stationary and the string has a horizontal component of velocity v to the left.

Speed of a Wave on a String

Consider a small-amplitude pulse transmitted along a horizontal string under a tension **F**. A stationary observer sees the pulse moving horizontally at a constant speed v and the string moving vertically but not horizontally (Fig. 16-13a).

If, however, the observer moves with the pulse at speed v, the pulse will appear to be stationary but the string will be moving horizontally as well as vertically. For the moving observer, the string has a horizontal velocity component v to the left (Fig. 16-13b). A point on the string that is instantaneously at the top of the wave form has *only* a horizontal component of velocity v. A small segment of the string centered on this point (shaded in the figure) follows an approximately circular path at speed v.

We can obtain an expression for the wave speed by analyzing the forces acting on this string segment of length ℓ and mass M, shown in an expanded form in Fig. 16-14. The two components of tension $F \sin\theta$ produce a resultant force in the radial direction, which, according to Newton's second law, produces centripetal acceleration v^2/R:

$$2(F\sin\theta) = Ma = \frac{Mv^2}{R}$$

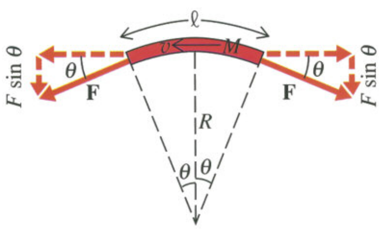


Fig. 16-14 An expanded view of the segment of string near the top of the pulse in Fig. 16-13.

The angle θ is small, and so $\sin\theta$ may be approximated by θ (as in Chapter 15, Fig. 15-11). We relate θ to ℓ and R, using Fig. 16-14:

$$\sin\theta \approx \theta = \frac{\ell/2}{R}$$

Substituting this expression for $\sin\theta$ into the equation above, we obtain

$$2F\frac{\ell/2}{R} = M\frac{v^2}{R}$$

Solving for v, we find

$$v = \sqrt{\frac{F\ell}{M}}$$

This expression indicates that the wave speed depends only on tension and on the mass per unit length, which we shall denote by the Greek letter μ:

$$\mu = \frac{M}{\ell} \qquad (16\text{-}2)$$

With this definition we may write v more concisely:

$$v = \sqrt{\frac{F}{\mu}} \qquad (16\text{-}3)$$

This equation predicts that if we increase the tension in a string the wave speed increases and if we replace the string by one having greater mass the wave speed decreases.

EXAMPLE 3 Transmitting a Wave Pulse on a String

Two people hold opposite ends of a 10.0 m long rope having a mass of 1.00 kg. The person at one end gives the rope a small upward jerk. How long is it before the person at the other end feels the jerk, if the rope is held with a tension of (a) 40.0 N; (b) 10.0 N?

SOLUTION **(a)** We first apply Eq. 16-3 to find the wave speed on the rope, which has a mass density of (1.00 kg)/(10.0 m), or 0.100 kg/m:

$$v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{40.0 \text{ N}}{0.100 \text{ kg/m}}}$$
$$= 20.0 \text{ m/s}$$

At this speed the wave will travel the 10.0 m length of the rope in a time interval

$$\Delta t = \frac{x}{v} = \frac{10.0 \text{ m}}{20.0 \text{ m/s}}$$
$$= 0.500 \text{ s}$$

(b) When the tension is reduced to 10.0 N, the speed decreases and the time interval increases:

$$v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{10.0 \text{ N}}{0.100 \text{ kg/m}}}$$
$$= 10.0 \text{ m/s}$$

The pulse is now slower and so it takes longer to transmit:

$$\Delta t = \frac{x}{v} = \frac{10.0 \text{ m}}{10.0 \text{ m/s}} = 1.00 \text{ s}$$

Speed of Sound

Table 16-1 gives the speed of sound in various media. Notice that sound travels considerably faster in solids and liquids than in gases. Unlike gas molecules, the molecules of solids and liquids are in constant contact with their neighbors. Consequently, these molecules respond more quickly to a wave pulse than do gas molecules, which interact only through occasional collisions.

In Chapter 12 we found that the rms speed of molecules in an ideal gas is given by Eq. 12-16:

$$v_{\text{rms}} = \sqrt{\frac{3kT}{m}}$$

where k is Boltzmann's constant, T is the absolute temperature, and m is the mass of a molecule. It is possible to use the laws of mechanics to prove that the speed of sound in an ideal gas is proportional to v_{rms}. The exact result of this derivation is

$$v = \sqrt{\frac{\gamma}{3}}\, v_{\text{rms}}$$

where γ equals 1.40 for diatomic gases like nitrogen and oxygen. Thus

$$v = \sqrt{\frac{1.40kT}{m}} \quad \text{(For a diatomic ideal gas)} \qquad (16\text{-}4)$$

The fact that wave speed is proportional to rms molecular speed in an ideal gas is certainly plausible, since the wave propagates as a result of the interaction of the gas molecules during collisions and the average time between collisions depends on the rms speed.

Table 16-1 Speed of sound

Medium	Speed (m/s)
Air (20° C)	344
Air (0° C)	332
Hydrogen (0° C)	1270
Water (20° C)	1480
Average body tissue (37° C)	1570
Aluminum	5100
Copper	3560
Iron	5130

EXAMPLE 4 The Wavelength of Sound Produced by a 512 Hz Tuning Fork

Suppose you strike a tuning fork that has a resonant frequency of 512 Hz. Find the speed and wavelength of the wave that propagates through the air if the air temperature is (a) 0° C; (b) 20.0° C.

SOLUTION **(a)** Air consists of approximately 80% nitrogen and 20% oxygen, and so the molecular mass of air is approximately $0.8(28) + 0.2(32) = 28.8$. This means that an average air molecule has a mass of 28.8 atomic mass units (u), where $1\ \text{u} = 1.66 \times 10^{-27}$ kg. Since both oxygen and nitrogen are diatomic molecules, we can apply Eq. 16-4:

$$v = \sqrt{\frac{1.40kT}{m}} = \sqrt{\frac{(1.40)(1.38 \times 10^{-23}\ \text{J/K})(273\ \text{K})}{(28.8)(1.66 \times 10^{-27}\ \text{kg})}}$$

$$= 332\ \text{m/s}$$

The wavelength is found when Eq. 16-1 ($v = \lambda f$) is applied:

$$\lambda = \frac{v}{f} = \frac{332\ \text{m/s}}{512\ \text{Hz}} = 0.648\ \text{m} = 64.8\ \text{cm}$$

(b) At 20.0° C, or 293 K, we find

$$v = \sqrt{\frac{1.40kT}{m}} = \sqrt{\frac{(1.40)(1.38 \times 10^{-23}\ \text{J/K})(293\ \text{K})}{(28.8)(1.66 \times 10^{-27}\ \text{kg})}}$$

$$= 344\ \text{m/s}$$

Sound travels somewhat faster through warmer air. Since f remains the same regardless of temperature, the wavelength increases:

$$\lambda = \frac{v}{f} = \frac{344\ \text{m/s}}{512\ \text{Hz}} = 0.672\ \text{m} = 67.2\ \text{cm}$$

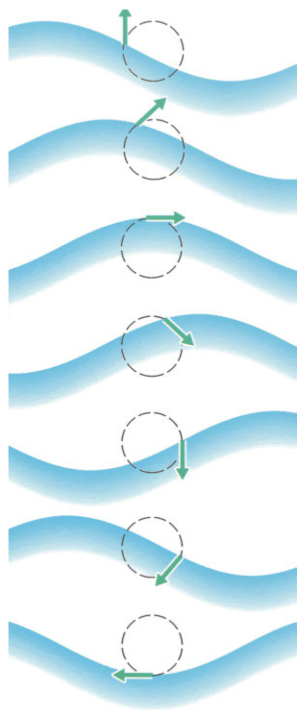


Fig. 16-15 As a deep-water wave passes, the water at the surface moves along a circular path whose radius is the amplitude of the wave.

Speed of Deep-Water Waves

Although water waves are easy to observe, they are relatively complex waves that are difficult to analyze in general. So we shall discuss only the special case of long-wavelength waves in deep water.

A wave is considered a **deep-water wave** if the depth of the water is much greater than the wavelength of the wave. In a deep-water wave each particle of water at the surface moves along a path in the vertical plane. If the wave is harmonic, a water particle's path is circular and a particle's speed is constant, as indicated in Fig. 16-15. As the wave propagates in a horizontal direction, the water undergoes both horizontal and vertical motion. Thus a water wave is neither transverse nor longitudinal. A body floating in the water will experience the same circular motion as the water. Suppose you are swimming in the ocean a good distance from shore. As a wave passes, you are first lifted, then pushed forward, then let down, and finally pushed back to your starting place, as indicated in the sequence of drawings in Fig. 16-15.

This circular motion is not limited to water at the surface; it extends to the water below, but the amplitude of the motion diminishes rapidly with depth. At a distance of 0.73λ below the surface, the amplitude is only 1% of the amplitude at the surface.

The speed of deep-water waves having wavelengths greater than about 10 cm is given by the following approximate expression:

$$v = \sqrt{\frac{g\lambda}{2\pi}} \quad \text{(deep-water waves; } \lambda \geq 10\ \text{cm)} \qquad (16\text{-}5)$$

One can derive this result by applying Bernoulli's equation (Problem 74). Notice that Eq. 16-5 implies that dispersion occurs for water waves; waves of different frequency (and therefore different wavelength) travel at different speeds.

EXAMPLE 5 Deep-Water Wave Speed

Find the speed of deep-water waves of wavelength (a) 2.00 m; (b) 5.00 m.

SOLUTION Applying Eq. 16-5, we find

(a) $$v = \sqrt{\frac{g\lambda}{2\pi}} = \sqrt{\frac{(9.80\ \text{m/s}^2)(2.00\ \text{m})}{2\pi}} = 1.77\ \text{m/s}$$

(b) $$v = \sqrt{\frac{(9.80\ \text{m/s}^2)(5.00\ \text{m})}{2\pi}} = 2.79\ \text{m/s}$$

16-3 Moving Sources and Observers: The Doppler Effect

Suppose you are standing beside a highway while cars pass by at high speed. As each car approaches, it produces a sound with a high frequency, or pitch. Just as the car passes, the pitch you hear drops significantly. Although the sound produced by the engine is unchanged, the frequency of the sound you hear is higher while the car is approaching than while it is moving away. This phenomenon is known as the **Doppler effect.** It occurs when either a source of sound or an observer of sound is in motion. Although the source produces a sound wave of a certain frequency f_S, the observed frequency f_O may be quite different.

Stationary Source, Moving Observer

Consider first the situation in which the source is stationary and the observer is moving. (All motion is measured relative to the medium of the sound wave.) The wavelength of the sound is determined by the frequency f_S of the source and the speed v of the sound through the medium:

$$\lambda = \frac{v}{f_S}$$

If the observer O is moving toward the source S at speed v_O as shown in Fig. 16-16, the observer passes more wavefronts per second than were emitted per second; that is, the observer hears a frequency that is greater than f_S. Since the wavefronts move *relative* to the observer at speed $v + v_O$, the observed frequency f_O will be this relative speed divided by λ:

$$f_O = \frac{v + v_O}{\lambda}$$

Substituting in this equation the value of λ from the preceding equation, we find

$$f_O = \frac{v + v_O}{v/f_S}$$

or

$$f_O = f_S\left(1 + \frac{v_O}{v}\right)$$

Next suppose that an observer is moving away from the source, as is observer O′ in Fig. 16-16. Now fewer wavefronts per second pass the observer; that is, a frequency lower than f_S is observed. The wavefronts move relative to the observer at a speed $v - v_O$. So the observed frequency is

$$f_O = \frac{v - v_O}{\lambda} = \frac{v - v_O}{v/f_S}$$

or

$$f_O = f_S\left(1 - \frac{v_O}{v}\right)$$

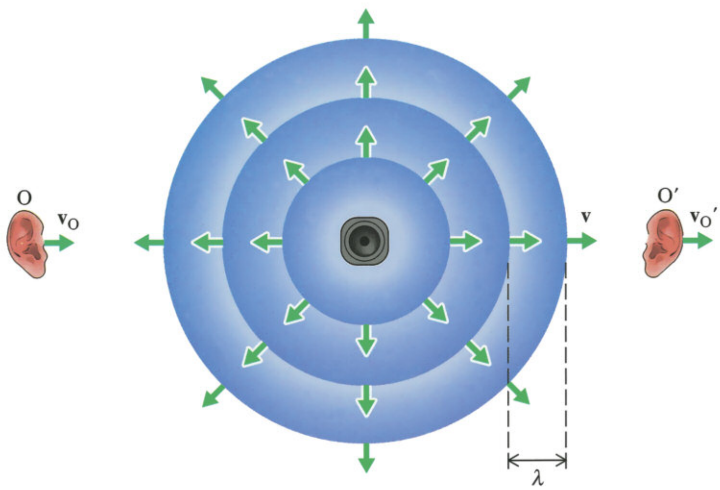


Fig. 16-16 Observers O and O′ hear sound produced by a stationary source. Observer O is moving toward the source at speed v_O, and observer O′ is moving away from the source at speed v_O'.

Summarizing the results for the observed frequency when the observer is moving either toward or away from the source, we have

$$f_O = f_S\left(1 \pm \frac{v_O}{v}\right) \quad \begin{array}{l}(+,\text{ observer moving toward source;}\\ -,\text{ observer moving away from source)}\end{array} \qquad (16\text{-}6)$$

Moving Source, Stationary Observer

Next we consider a moving source and a stationary observer. Fig. 16-17 shows two wavefronts produced by a source that is moving to the right at constant velocity v_S. At $t = 0$, the source was at point S, at which instant it emitted the larger wavefront shown in the figure—a spherical surface centered at S. At $t = T = 1/f_S$, the source emitted from point S′ the smaller wavefront shown in the figure—a spherical surface centered at S′, a distance $v_S T$ to the right of S. Adjacent wavefronts in front of the moving source are squeezed together, whereas adjacent wavefronts behind the moving source are spread out. The observed wavelengths are respectively $\lambda - v_S T$ and $\lambda + v_S T$. Since the observer is at rest with respect to the medium, the relative speed of the wavefronts is just the speed v of waves through the medium. The observed frequency in front of the source is

$$f_O = \frac{v}{\lambda - v_S T}$$

where λ may be expressed as v/f_S:

$$f_O = \frac{v}{v/f_S - v_S/f_S}$$

or

$$f_O = \frac{f_S}{1 - v_S/v}$$

Behind the moving source, the observed frequency is $v/(\lambda + v_S T)$, and

$$f_O = \frac{f_S}{1 + v_S/v}$$

Summarizing these results for a moving source, we have

$$f_O = \frac{f_S}{1 \mp v_S/v} \quad \begin{array}{l}(-\text{, source moving toward observer;}\\ +\text{, source moving away from observer)}\end{array} \qquad (16\text{-}7)$$

If both the observer and the source are moving through the medium, factors from both Eq. 16-6 and Eq. 16-7 are simultaneously present. We find in general

$$f_O = f_S\left(\frac{1 \pm v_O/v}{1 \mp v_S/v}\right) \quad \begin{array}{l}\text{(upper signs if toward;}\\ \text{lower signs if away)}\end{array} \qquad (16\text{-}8)$$

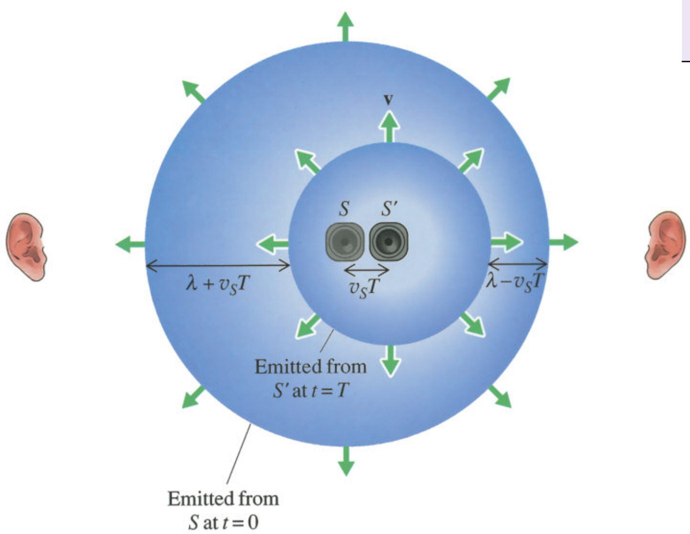


Fig. 16-17 A source of sound moves at speed v_S away from one observer and toward another observer. Wavefronts move at the speed of sound v.

EXAMPLE 6 Listening to the Whistle of an Approaching Train

A train travels parallel to a highway at a speed of 35.0 m/s. A car traveling on the highway at 30.0 m/s in the opposite direction is approaching the train. The driver of the car hears the train's whistle at a frequency of 650 Hz. (Fig. 16-18).
(a) What is the frequency of the whistle as heard on the train?
(b) After the train passes, what is the frequency of the whistle as heard by the car's driver? Use 344 m/s as the speed of sound.

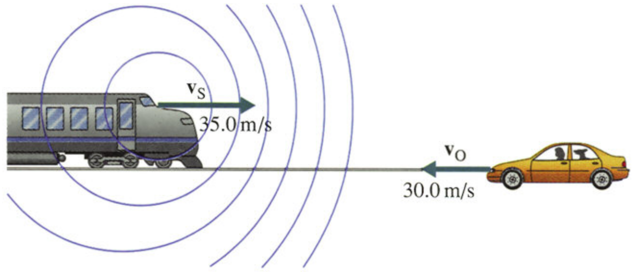


Fig. 16-18

SOLUTION **(a)** We apply Eq. 16-8 with the two upper signs, since the observer and the source are moving toward each other:

$$f_O = f_S\left(\frac{1 + v_O/v}{1 - v_S/v}\right)$$

$$650 \text{ Hz} = f_S\left(\frac{1 + (30.0 \text{ m/s})/(344 \text{ m/s})}{1 - (35.0 \text{ m/s})/(344 \text{ m/s})}\right)$$

$$650 \text{ Hz} = 1.21 f_S$$

or

$$f_S = \frac{650 \text{ Hz}}{1.21} = 537 \text{ Hz}$$

(b) After the source passes, observer and source are moving away from each other. So we use the lower signs in Eq. 16-8 to find the frequency heard by the driver:

$$f_O = f_S\left(\frac{1 - v_O/v}{1 + v_S/v}\right)$$

$$= (537 \text{ Hz})\left(\frac{1 - (30.0 \text{ m/s})/(344 \text{ m/s})}{1 + (35.0 \text{ m/s})/(344 \text{ m/s})}\right)$$

$$= 445 \text{ Hz}$$

As the train passes, the driver hears a dramatic drop in frequency, from 650 Hz to 445 Hz.

The Electromagnetic Doppler Effect

The Doppler effect is not limited to sound waves. All waves experience a similar effect. In particular, all electromagnetic waves, including visible light, undergo a Doppler shift when there is relative motion of observer and source. However, because of the unique nature of electromagnetic waves, the analysis of their Doppler shift is different from the analysis we used for sound waves.

Here we simply state without proof the relationship between the source frequency f_S and the observed frequency f_O. For electromagnetic waves, it is only the *relative* velocity of observer and source that counts. Denoting the relative speed by v and the speed of light by c, the following equation provides a good approximation to the observed frequency, so long as v is much less than c, where $c = 3.00 \times 10^8$ m/s:

$$f_O \approx f_S\left(1 \pm \frac{v}{c}\right) \quad (+, \text{toward}; -, \text{away}) \; (v << c) \qquad (16\text{-}9)$$

The Doppler effect is important in astronomy, where it is used to determine the speed of a star emitting light that is observed at a frequency f_O shifted somewhat from the frequency f_S that would be emitted by the same kind of source if it were stationary.

The electromagnetic Doppler effect is also utilized in police radar units. Electromagnetic radiation of radar frequency is reflected from a moving car back to the radar unit. The speed of the car is determined when the Doppler shift of the radar frequency is observed.

Supersonic Speeds

So far we have considered only sources of sound moving at speeds less than the speed of sound, or at "subsonic" speeds. If the source moves at a "supersonic" speed, that is, faster than the speed of sound, a region of intense sound is created. This is known as a **sonic boom.** The effect is caused by the concentration of sound energy in a relatively small region of space. Wherever wavefronts are very close together, there is a concentration of energy. As these closely spaced wavefronts sweep past an observer, the observed intensity of the sound is quite large.

Fig. 16-19a shows a source moving to the right at a subsonic speed. Notice that there is some concentration of energy in front of the source. Fig. 16-19b shows a source moving slightly faster than the speed of sound, and Fig. 16-19c shows a source moving much faster than the speed of sound. In all three figures, the region of concentrated sound energy is inside the dashed lines. In Fig. 16-19c the concentrated region is along the surface of a cone.

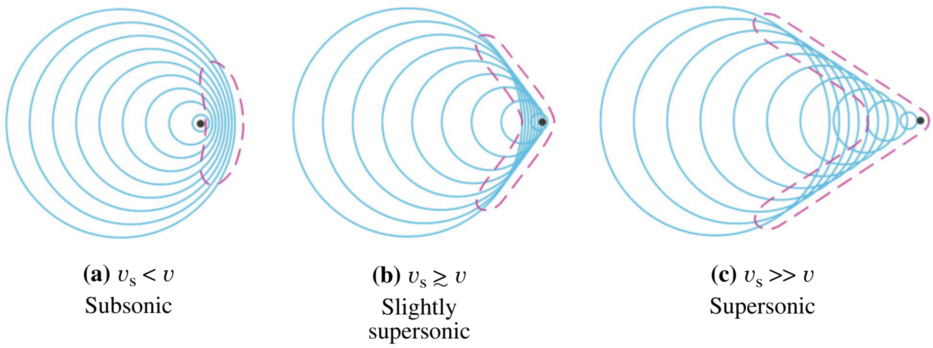


Fig. 16-19 A source of sound travels at a speed v_S **(a)** less than the speed of sound; **(b)** slightly greater than the speed of sound; **(c)** much greater than the speed of sound. Dashed lines show regions where energy is most concentrated.

Fig. 16-20 shows that the surface of this cone makes an angle θ with the direction of motion, where

$$\sin\theta = \frac{v}{v_S} \tag{16-10}$$

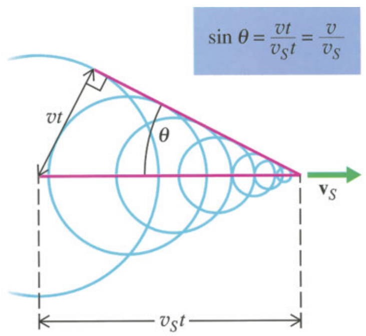


Fig. 16-20 When a source moves at a speed v_S greater than the speed of sound v, sound energy is concentrated along the surface of a cone. The surface makes an angle θ with the direction of motion.

The inverse of this ratio, v_S/v, is called the "mach number." For example, if an object moves at mach 2, its speed is twice the speed of sound, or about 680 m/s.

As the surface of the cone passes an observer, a loud sound of short duration is heard—a sonic boom. Notice that this sonic boom occurs not just as the "sound barrier" is broken, as many people erroneously believe. It is produced for as long as the source moves at supersonic speed, and anyone in the path of the conical edge of the wavefront will hear it. Fig. 16-21 shows a supersonic plane at equal time intervals. The surface of the cone sweeps along the ground at the same speed as the plane.

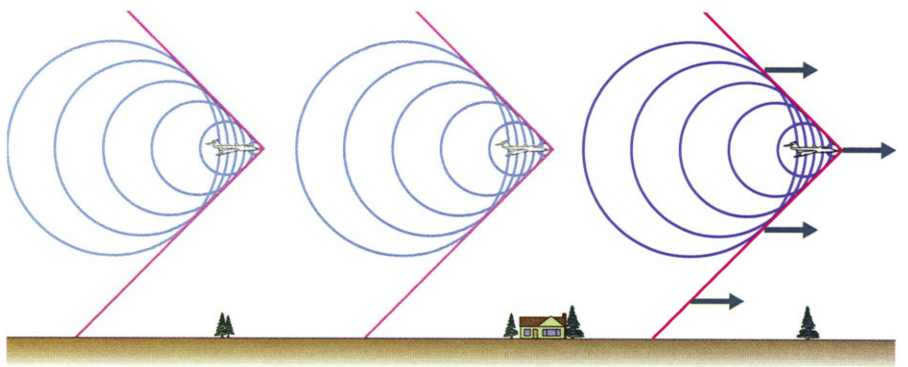


Fig. 16-21 A conical surface of concentrated sound energy sweeps over the ground as a supersonic plane passes overhead.

The crack of a whip is the shock wave, or sonic boom, produced by the tip of the whip moving at supersonic speed. The sound of a bullet is a similar effect. When a boat moves through water at a speed greater than the speed of wave propagation, the pattern of waves in its wake is a two-dimensional version of the wave pattern in a sonic boom (Fig. 16-22). The front edge of the wake is the region with the greatest concentration of energy.

Fig. 16-22 The passing of the leading edge of a boat's wake is the water wave equivalent of a sonic boom and is created when a boat moves through water at a speed greater than that of the waves it produces.

16-4 Power and Intensity; the Decibel Scale

Waves sometimes transmit large amounts of energy in short intervals of time. For example, when Hurricane Iniki hit Hawaii in 1992, the waves spawned by the hurricane carried enough energy to cause great destruction.

In this section we shall study the rate at which energy is transmitted in various waves, that is, the power carried by a wave (Fig. 16-23).

Fig. 16-23 Water waves sometimes carry large amounts of energy.

Harmonic Waves on a String

Consider first the power transmitted by a harmonic wave on a string. The power P is defined as the energy per unit time transmitted past a given particle on the string. In Fig. 16-24 a particle at point O is about to experience the passage of one cycle of a harmonic wave. At the end of one cycle, the wave form has passed O. So during a time interval equal to the period of the motion, all the energy contained in a segment of the string of length λ has been transmitted past O. The power transmitted equals the energy E contained in one wavelength divided by the period T, the time during which the energy passes:

$$P = \frac{E}{T}$$

The energy E in one wavelength can be calculated from the fact that the motion of each particle of the string is SHM, like the motion of a mass m on a spring of force constant k. Using Eq. 15-15 $\left(T = 2\pi\sqrt{\frac{m}{k}}\right)$, we can solve for k in terms of m and the period T:

$$k = \frac{4\pi^2 m}{T^2}$$

The mass m is the mass of one wavelength of the string. Using Eq. 16-2 $\left(\mu = \frac{m}{L} = \frac{m}{\lambda}\right)$, we find

$$m = \mu\lambda$$

We insert the two preceding equations into the expression for the total energy of a mass on a spring oscillating with amplitude A (Eq. 15-16: $E = \frac{1}{2}kA^2$) to obtain

$$E = \frac{2\pi^2\mu\lambda A^2}{T^2}$$

To find the power, we divide E by T:

$$P = \frac{E}{T} = \frac{2\pi^2\mu\lambda A^2}{T^3}$$

Substituting $T = \frac{1}{f}$ and $\lambda = \frac{v}{f}$, we obtain

$$P = 2\pi^2\mu v f^2 A^2 \qquad (16\text{-}11)$$

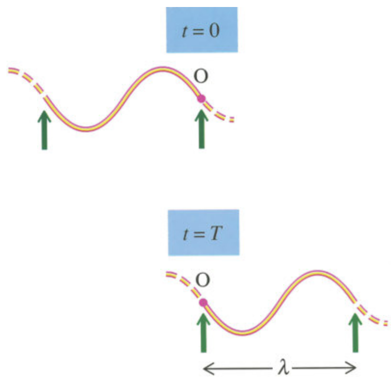


Fig. 16-24 The kinetic and potential energy contained in one wavelength of string at $t = 0$ is transmitted through the particle O in the time interval T.

EXAMPLE 7 Power Transmitted by a Wave on a String

A 10 Hz harmonic wave of amplitude 5.0 cm travels at 30 m/s along a string having a mass density of 0.020 kg/m. Find the power transmitted.

SOLUTION Applying Eq. 16-11, we find

$$\begin{aligned} P &= 2\pi^2 \mu v f^2 A^2 \\ &= 2\pi^2 (0.020 \text{ kg/m})(30 \text{ m/s})(10 \text{ Hz})^2 (0.050 \text{ m})^2 \\ &= 3.0 \text{ W} \end{aligned}$$

Harmonic Sound Waves

In contrast to a wave on a string, which transmits energy along a line, a sound wave spreads energy over a volume of space. We shall find an expression for the rate at which sound energy is transported from one region of space to another, passing through a cross-sectional area perpendicular to the direction of motion.

(a) (b)

Fig. 16-25 **(a)** A plane wave passes through an open window that is oriented perpendicular to the direction of motion. Sound energy passes into the room through the window of area $\mathscr{A}$. **(b)** A spherical wave passes through a cross-sectional area $\mathscr{A}$, which is part of a spherical surface.

Motion of a sound wave through a surface area is illustrated in Fig. 16-25 for both plane and spherical waves. If we apply to a harmonic sound wave the same reasoning used to obtain an expression for the power carried by a harmonic wave on a string, we obtain the same result, Eq. 16-11:

$$P = 2\pi^2 \mu v f^2 A^2$$

where μ, the mass per unit length of the medium, is the product of the density ρ of the medium and the cross-sectional area $\mathscr{A}$, as shown in Fig. 16-26 for a plane wave:

$$\mu = \rho \mathscr{A}$$

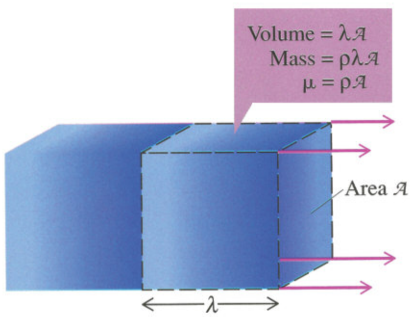


Fig. 16-26 Finding the mass per unit length of the medium for a sound wave

Substituting this value for μ into the preceding equation, we obtain

$$P = 2\pi^2 \rho \mathscr{A} v f^2 A^2 \tag{16-12}$$

Often the concentration of power in a sound wave is of more importance than the total power; that is, we may wish to know how much power is transmitted per unit area of the wave. We define the power divided by the cross-sectional area perpendicular to the direction of wave motion to be the **intensity** I of the sound wave:

$$I = \frac{P}{\mathscr{A}} \tag{16-13}$$

Combining Eqs. 16-12 and 16-13, we obtain an expression for the intensity of a harmonic sound wave of amplitude A and frequency f, moving at speed v through a fluid of density ρ:

$$I = 2\pi^2 \rho v f^2 A^2 \tag{16-14}$$

The Decibel Scale

The loudness of a sound is related to its intensity. Normally the human ear is capable of hearing sounds of even very low intensity over a wide range of frequencies. The greatest sensitivity of the ear is at frequencies of a few thousand hertz. The most sensitive individuals can hear sounds having intensities as low as about 10^{-12} W/m^2 at a frequency of about 4000 Hz. We shall take this intensity as our reference level—the lowest sound intensity perceptible—and denote it by I_0:

$$I_0 = 1.00 \times 10^{-12} \text{ W/m}^2 \tag{16-15}$$

The sounds we commonly hear are somewhere in the range I_0 to $10^{12} I_0$. When sound intensity reaches about 1 W/m^2, or $10^{12} I_0$, the sound becomes painful. This level of intensity is typical of the sound near the loudspeakers at a rock concert.

Two sounds that differ significantly in loudness will have intensities whose ratio will be some power of 10. For example, the rustling of leaves produces sound nearby of intensity roughly $10 I_0$. A whisper, which is perceived as just a little louder than the leaves, might have an intensity of about $10^2 I_0$. It is therefore useful to define a logarithmic scale for measuring sounds. The **intensity level,** denoted by the Greek letter β (beta), is defined as

$$\beta = 10 \log\left(\frac{I}{I_0}\right) \tag{16-16}$$

Intensity level is a measure of the intensity of sound relative to the reference intensity I_0. The number β is expressed as decibels, abbreviated dB. The lowest audible sound has an intensity I_0 and an intensity level

$$\beta = 10 \log\left(\frac{I_0}{I_0}\right) = 10 \log 1 = 0$$

As mentioned above, the threshold of pain occurs at an intensity of 1 W/m^2, which corresponds to an intensity level

$$\beta = 10 \log\left(\frac{I}{I_0}\right) = 10 \log\left(\frac{1 \text{ W/m}^2}{10^{-12} \text{ W/m}^2}\right)$$

$$= 10 \log 10^{12} = (10)(12)$$

$$= 120 \text{ dB}$$

Audibility curves in Fig. 16-27 show the minimum audible intensity levels for individuals with very sensitive, average, and severely handicapped hearing. Notice that 1% of the population cannot hear sounds below about 70 dB.

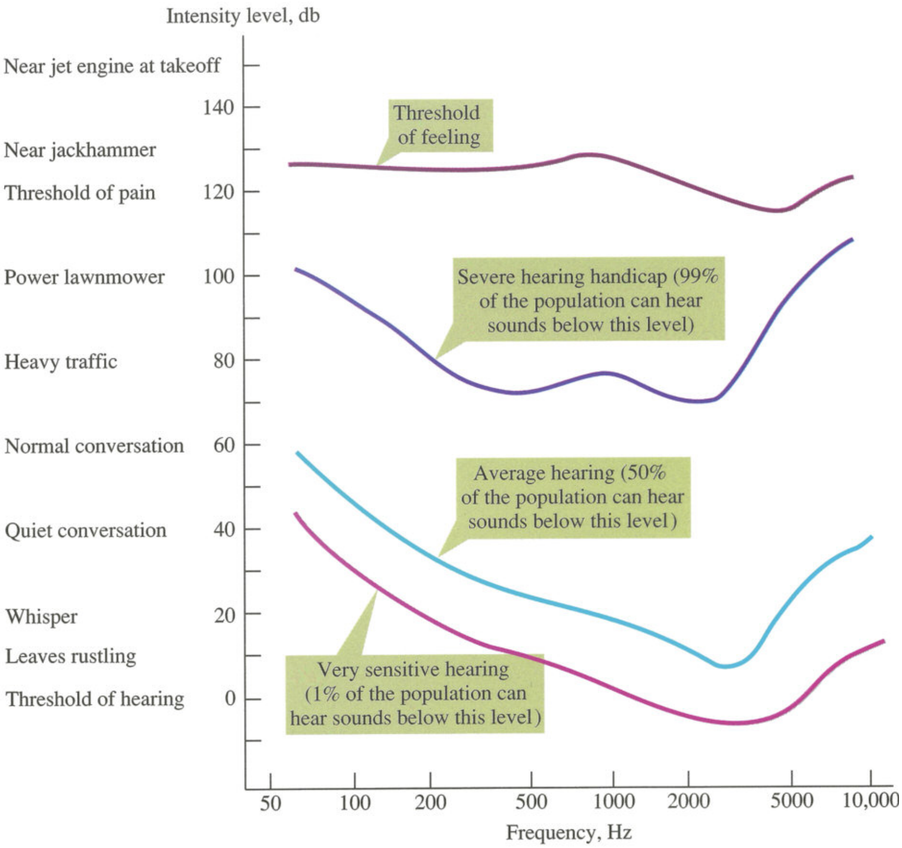


Fig. 16-27 Audibility curves.

EXAMPLE 8 Decibels Decrease as Distance from a Source of Sound Increases

A person 2.00 m away from you and talking in a normal voice produces 60.0 dB sound at your ear. (a) Assuming that the sound propagates equally in all directions, how much power is delivered by the person talking? (b) What is the intensity level at a distance of 6.00 m?

SOLUTION **(a)** Solving for I in Eq. 16-16:

$$\beta = 10 \log\left(\frac{I}{I_0}\right),$$

we find $I = I_0 10^{\beta/10}$

$$= (1.00 \times 10^{-12}\ \text{W/m}^2)(10^{60.0/10})$$

$$= 1.00 \times 10^{-6}\ \text{W/m}^2$$

If we assume that the sound propagates equally in all directions, the intensity of sound is the same everywhere on a spherical surface of radius $r = 2.00$ m. The total power is spread over this surface and is equal to the product of the intensity and the surface area $\mathscr{A} = 4\pi r^2$:

$$P = I\mathscr{A} = I4\pi r^2$$

$$= (1.00 \times 10^{-6}\ \text{W/m}^2)(4\pi)(2.00\ \text{m})^2 = 5.03 \times 10^{-5}\ \text{W}$$

(b) At $r = 6.00$ m, the distance has tripled, and since $\mathscr{A} \propto r^2$, the surface area is 9 times as great as before. Thus the intensity is reduced by a factor of 9:

$$I' = \frac{I}{9} = \frac{1.00 \times 10^{-6}\ \text{W/m}^2}{9}$$

$$= 1.11 \times 10^{-7}\ \text{W/m}^2$$

And the new intensity level β' is found from the definition:

$$\beta' = 10 \log\left(\frac{I}{I_0}\right)$$

$$= 10 \log\left(\frac{1.11 \times 10^{-7}\ \text{W/m}^2}{1.00 \times 10^{-12}\ \text{W/m}^2}\right)$$

$$= 50.5\ \text{dB}$$

EXAMPLE 9 Small Amplitude and Large Amplitude Sound Waves

(a) Find the amplitude of vibration of air molecules at 20.0° C at a frequency of 1000 Hz for sounds of intensity level 0 dB and 160 dB. (b) Repeat the calculation for a frequency of 100 Hz.

SOLUTION **(a)** Solving for A in Eq. 16-14 ($I = 2\pi^2 \rho v f^2 A^2$), we find

$$A = \sqrt{\frac{I}{2\pi^2 \rho v f^2}}$$

A 0 dB sound corresponds to an intensity $I_0 = 1.00 \times 10^{-12}$ W/m^2. We find from Table 10-1 that the density of air at 20.0° C is 1.20 kg/m^3 and from Table 16-1 that the speed of sound at this temperature is 344 m/s. Inserting these values into the equation above, we find

$$A = \sqrt{\frac{1.00 \times 10^{-12}\ \text{W/m}^2}{2\pi^2 (1.20\ \text{kg/m}^3)(344\ \text{m/s})(1.00 \times 10^3\ \text{Hz})^2}}$$

$$= 1.11 \times 10^{-11}\ \text{m}$$

This is an extremely small displacement, about $\frac{1}{10}$ the diameter of the smallest atom, and yet the human ear is capable of hearing such sounds.

A 160 dB sound is 10^{16} times as intense as a 0 dB sound. Since $A \propto \sqrt{I}$, this means that the new amplitude A' is $\sqrt{10^{16}}$ times A:

$$A' = \sqrt{10^{16}}\, A = (10^8)(1.11 \times 10^{-11}\ \text{m})$$

$$= 1.11 \times 10^{-3}\ \text{m}$$

(b) Notice that the amplitude is proportional to $1/f$. Thus, when the frequency is reduced to 100 Hz, or one tenth of its former value, the amplitude is 10 times as great. At 0 dB, we have

$$A = (10.0)(1.11 \times 10^{-11}\ \text{m})$$

$$= 1.11 \times 10^{-10}\ \text{m}$$

And at 160 dB, we find

$$A = (10.0)(1.11 \times 10^{-3}\ \text{m})$$

$$= 1.11 \times 10^{-2}\ \text{m}$$

$$= 1.11\ \text{cm}$$

So for very low-frequency, high-intensity sounds, the displacement of air molecules is quite large. This effect can be dramatically demonstrated. A candle flame held near a loudspeaker will flicker whenever the loudspeaker emits very loud, low-frequency sounds (Fig. 16-28).

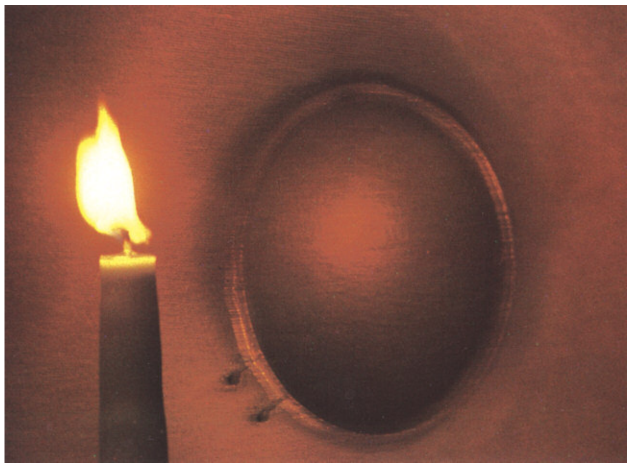


Fig. 16-28

16-5 Time Dependence of the Displacement of a Particle of the Medium

Next we shall obtain an expression for the time dependence of a wave's displacement at a given point. We shall consider only harmonic one-dimensional waves and harmonic plane waves, so that spatial variations occur only in one direction, which we take to be the x-axis. Suppose the wave source is located at the origin ($x = 0$) and oscillates in SHM. If we assume zero displacement when $t = 0$, the displacement y at the origin is given by Eq. 15-1:

$$y = A \sin\left(2\pi \frac{t}{T}\right)$$

At any point along the wave, the motion of any particle of the medium is of the same type, that is, SHM of period T and amplitude A. However, the particle motion at any point is delayed by the time it takes for the wave to travel to that point. If the wave travels in the positive x direction at speed v, the time delay $\Delta t = \dfrac{x}{v}$. So the particle motion at point x at time t is the same as the motion at the origin at time $t - \dfrac{x}{v}$, as given by the equation

$$y = A \sin\left(2\pi \frac{t - x/v}{T}\right) = A \sin\left[2\pi\left(\frac{t}{T} - \frac{x}{vT}\right)\right]$$

or, since $vT = \dfrac{v}{f} = \lambda$, we may express this result

$$y = A \sin\left[2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)\right] \qquad (+x \text{ direction}) \qquad (16\text{-}17)$$

The following example illustrates how this expression represents wave motion along the positive x-axis. For motion along the negative x-axis, we replace v by $-v$ in the expression for the time delay ($t - x/v \rightarrow t - x/{-v} = t + x/v$), which leads to the expression

$$y = A \sin\left[2\pi\left(\frac{t}{T} + \frac{x}{\lambda}\right)\right] \qquad (-x \text{ direction}) \qquad (16\text{-}18)$$

EXAMPLE 10 Snapshots of a Waveform

A wave source at the origin oscillates in SHM at a frequency of 5.0 Hz and with an amplitude A. A plane wave travels in the $+x$ direction at a speed of 10 m/s. Find an expression for the displacement y at any point x at $t = 0$, $t = 0.025$ s, and $t = 0.050$ s. Graph y versus x for each of these times. (These three graphs correspond to three "snapshots" of the waveform at equal time intervals.)

SOLUTION First we must calculate T and λ and then apply Eq. 16-17:

$$T = \frac{1}{f} = \frac{1}{5.0\text{ Hz}} = 0.20\text{ s}$$

$$\lambda = \frac{v}{f} = \frac{10\text{ m/s}}{5.0\text{ Hz}} = 2.0\text{ m}$$

$$y = A\sin\left[2\pi\left(\frac{t}{0.20} - \frac{x}{2}\right)\right]$$

where t is in s and x is in m.

At $t = 0$: $y = A\sin(-\pi x)$

At $t = 0.025$ s: $y = A\sin\left(\frac{\pi}{4} - \pi x\right)$

At $t = 0.050$ s: $y = A\sin\left(\frac{\pi}{2} - \pi x\right)$

The graphs of y as a function of x are given in Fig. 16-29. These graphs show a waveform moving to the right at a speed of 10 m/s.

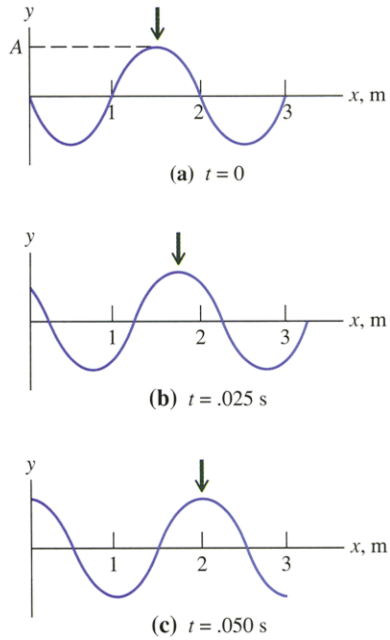


Fig. 16-29

16-6 Superposition of Waves; Beats; Standing Waves

Superposition Principle

When two (or more) waves move through a medium, the net effect on the medium is a wave whose displacement at any point is found by adding the two (or more) separate wave displacements. This principle, called the **superposition principle,** is illustrated in Fig. 16-30 by two pulses traveling in opposite directions along a spring. When they cross, what is seen is a displacement of the medium that equals the sum of the displacements of the two pulses.

The superposition principle applies to sound waves. For example, suppose that 10 people are in a room. If only one person is talking, the sound wave will have—at some time and at a given point in the room—a certain displacement y_1. If that person stops talking and a second person begins talking, there will be a displacement y_2 at that same point; a third person speaking alone would produce a displacement y_3 at the point, and so forth. When all 10 speak at once, the displacement y is the sum of the individual displacements: $y = y_1 + y_2 + \ldots + y_{10}$. (Despite the presence of all these waves producing an additive net effect, it is a remarkable fact that the human ear is often able to pick out individual voices in such a situation—at a party, for example.)

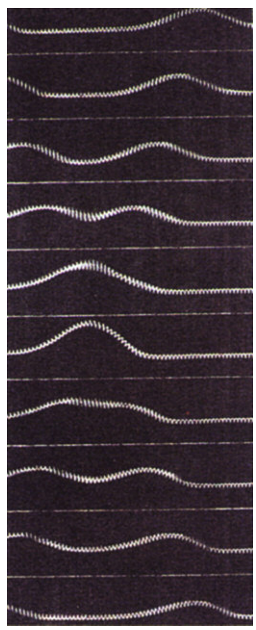


Fig. 16-30 Superposition of two wave pulses.

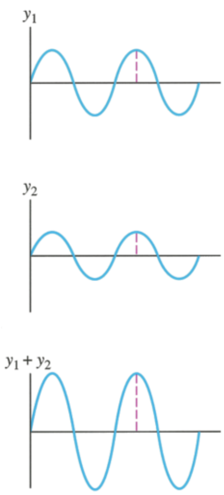


Fig. 16-31 Constructive interference.

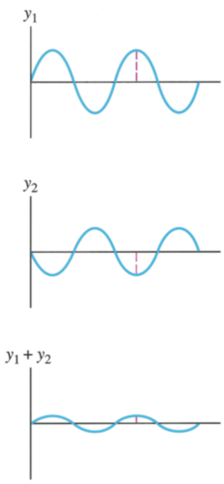


Fig. 16-32 Destructive interference.

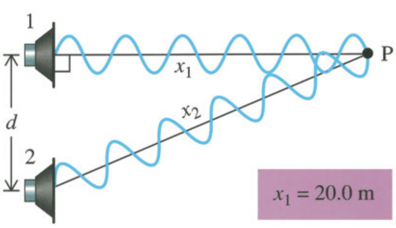


Fig. 16-33 Sound waves from two stereo speakers interfere destructively at P because sound from speaker 2 travels ½ wavelength farther than sound from speaker 1.

Constructive and Destructive Interference

Suppose that two harmonic waves of the same frequency both travel in the positive x direction through some medium. The waves may differ in amplitude and in initial phase angle. The two displacements are

$$y_1 = A_1 \sin\left[2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)\right] \tag{16-19}$$

$$y_2 = A_2 \sin\left[2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right) + \phi\right] \tag{16-20}$$

There are two important special cases: $\phi = 0$ and $\phi = \pi$.

When $\phi = 0$, the two waves are said to be "in phase." We find the resulting wave displacement either graphically, as indicated in Fig. 16-31, or algebraically, by adding the two preceding expressions, with $\phi = 0$:

$$\begin{aligned} y &= y_1 + y_2 \\ &= A_1 \sin\left[2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)\right] + A_2 \sin\left[2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)\right] \\ &= (A_1 + A_2) \sin\left[2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)\right] \end{aligned}$$

The total displacement y has the same dependence on x as the two individual waves have. The total amplitude is the sum of the two amplitudes A_1 and A_2. This effect is called **constructive interference.** When $\phi = \pi$, that is, when the two waves are 180° out of phase, they tend to cancel each other. The effect, called **destructive interference,** is seen graphically in Fig. 16-32, or we can find it by adding Eqs. 16-19 and 16-20 after setting $\phi = \pi$ in Eq. 16-20:

$$y = y_1 + y_2 = A_1 \sin\left[2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)\right] + A_2 \sin\left[2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right) + \pi\right]$$

But since $\sin(\theta + \pi) = -\sin\theta$, we may express this as

$$y = A_1 \sin\left[2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)\right] - A_2 \sin\left[2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)\right]$$

or

$$y = (A_1 - A_2) \sin\left[2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)\right]$$

Here the amplitude is the difference in the two amplitudes A_1 and A_2. If A_1 and A_2 are equal, the destructive interference of these waves results in an amplitude of zero. In this case, the two waves cancel. The presence of these two waves results in no wave at all!

Difference in Path Length

Interference effects often arise when a phase difference results from different distances to the sources of the waves. Consider a point P located a distance x_1 from source 1 and a distance x_2 from source 2 (Fig. 16-33). We assume that each wave has the same amplitude A, the same frequency f, and the same initial phase angle $\phi_0 = 0$. Sources that either are in phase or have a constant phase difference are said to be **coherent.** Since the waves move through the same medium at the same speed and with the same frequency, it follows that they also have the same wavelength $\lambda = \frac{v}{f}$. The respective displacements at point P are

$$y_1 = A \sin\left[2\pi\left(\frac{t}{T} - \frac{x_1}{\lambda}\right)\right]$$

$$y_2 = A \sin\left[2\pi\left(\frac{t}{T} - \frac{x_2}{\lambda}\right)\right]$$

The difference in the phase angles is

$$\Delta\phi = 2\pi\,\frac{x_2}{\lambda} - 2\pi\,\frac{x_1}{\lambda}$$

or, letting $\Delta x = x_2 - x_1$,

$$\Delta\phi = 2\pi\,\frac{\Delta x}{\lambda} \qquad (16\text{-}21)$$

If there is no difference in the path length ($\Delta x = 0$), then $\Delta\phi = 0$ and we have constructive interference. Constructive interference can also occur if Δx equals any integer multiple of λ, for then the phase difference is a multiple of 2π, and this is equivalent to no phase difference because it leaves the sine unchanged [$\sin(\theta + 2\pi n) = \sin\theta$]. So constructive interference occurs when the path-length difference is an integer multiple of λ:

$$\Delta x = 0, \lambda, 2\lambda, 3\lambda, \ldots \qquad \text{(constructive interference)} \qquad (16\text{-}22)$$

Destructive interference occurs whenever $\Delta\phi = \pi$, or 3π, or 5π, ... From Eq. 16-21, we see that such phase differences result when Δx takes on the following values:

$$\Delta x = \frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2}, \ldots \qquad \text{(destructive interference)} \qquad (16\text{-}23)$$

EXAMPLE 11 Stereo Speakers Producing Sounds that Cancel Each Other Out

Two stereo speakers are connected to an oscillator that causes the speakers to produce identical harmonic sound waves of wavelength 20.0 cm and frequency 1720 Hz. The two sources are coherent; that is, they oscillate in phase. Let P be a point 20.0 m from the first speaker, as shown in Fig. 16-33. How far from speaker 1 must speaker 2 be placed for there to be destructive interference at P?

SOLUTION For destructive interference to occur, the minimum spacing between the speakers must be such that there is a difference in the path lengths x_1 and x_2 equal to half a wavelength, as indicated in the figure:

$$x_2 - x_1 = \frac{\lambda}{2}$$

$$x_2 = x_1 + \frac{\lambda}{2} = 20.0\text{ m} + \frac{0.200\text{ m}}{2} = 20.1\text{ m}$$

Applying the Pythagorean theorem in Fig. 16-33, we find

$$d = \sqrt{x_2^2 - x_1^2} = \sqrt{(20.1\text{ m})^2 - (20.0\text{ m})^2} = 2.00\text{ m}$$

If the speakers are placed 2.00 m apart, there is no sound at P. A listener at P hears nothing. If the listener moves 1.00 m laterally from P so that she is equidistant from the two speakers, sound is heard. In this case the two waves interfere constructively, producing sound with an amplitude equal to twice the amplitude produced by one speaker alone. And since intensity is proportional to the square of the amplitude, the intensity of the sound is four times as great as the intensity produced by one speaker. A pattern of alternating constructive and destructive interference is produced in the entire region of space around the speakers. This two-source interference pattern has an analog in optics, which we shall study in Chapter 26.

Beats

When two waves have slightly different frequencies, interference at any point in space can alternate between constructive and destructive. Suppose, for example, you are tuning a guitar by comparing the frequencies of the sounds produced by two of its strings. When the frequencies are close but not equal, you hear beats—a sound that varies periodically in intensity—a throbbing or pulsing variation from loud to weak to loud. Fig. 16-34 shows the displacement of two harmonic waves of slightly different frequencies and periods (T_1 and T_2) as a function of time. The bottom of the figure shows the sum of the two waves, which represents the wave disturbance when both waves are simultaneously present. This resultant wave shows a periodic variation in amplitude.

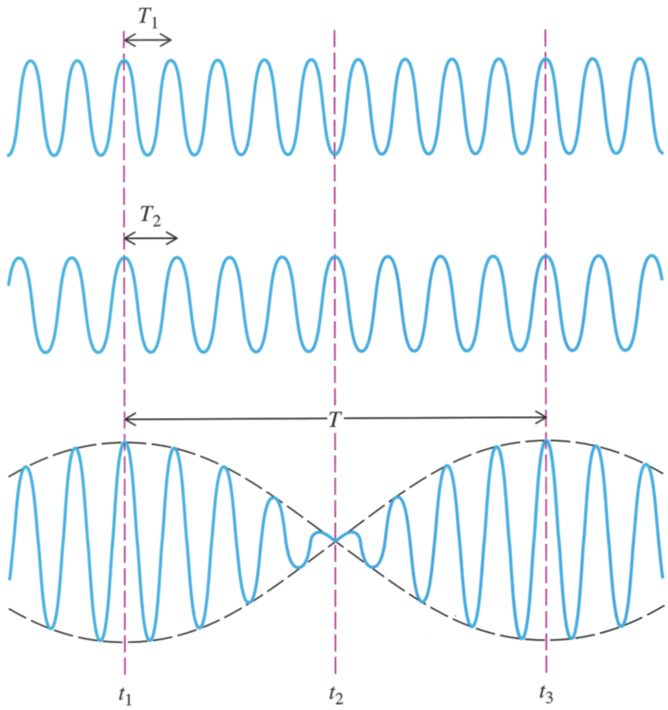


Fig. 16-34 Interference of two sound waves of slightly different frequencies produces beats—a sound that varies in intensity.

Notice that at time t_1 the waves interfere constructively but at time t_2 they interfere destructively. At time t_3 they interfere constructively again. The resultant wave's period $T = t_3 - t_1$ can be expressed in terms of the periods T_1 and T_2 of the interfering waves. Constructive interference occurs when both waves are at a peak. Each wave must undergo a number of complete cycles before constructive interference can occur again. The wave with a shorter period (T_1) must undergo one more cycle than the wave with the longer period (T_2). Thus

$$T = nT_2$$

and

$$T = (n + 1)T_1$$

Equating these expressions and solving for n, we find

$$n = \frac{T_1}{T_2 - T_1}$$

and, since $T = nT_2$,

$$T = \frac{T_1 T_2}{T_2 - T_1}$$

This result is better expressed in terms of the beat frequency f_B, the inverse of the beat period T. Taking the inverse of the equation above, we find

$$f_B = \frac{1}{T} = \frac{T_2 - T_1}{T_1 T_2} = \frac{1}{T_1} - \frac{1}{T_2}$$

or

$$f_B = f_1 - f_2 \tag{16-24}$$

For example, if two guitar strings have frequencies of 440 Hz and 445 Hz, when both strings are plucked together, you hear beats at a beat frequency of 5 Hz; that is, the amplitude and intensity of the sound vary from loud to soft to loud with a period of $\frac{1}{5}$ second.

Standing Waves on a String

Suppose a harmonic wave is generated by an oscillator at the left end of a stretched string, with the right end held fixed. Fig. 16-35 shows the string at equal time intervals. The wave travels to the right, reaches the right end, and is reflected back. As soon as the reflected wave begins to propagate from the right end, it interferes with the incident wave. As indicated in Fig. 16-35, the superposition of the incident wave (dotted line) and the reflected wave (dashed line) produces, near the right end, a wave that is different from the original traveling wave (shaded part of figure).

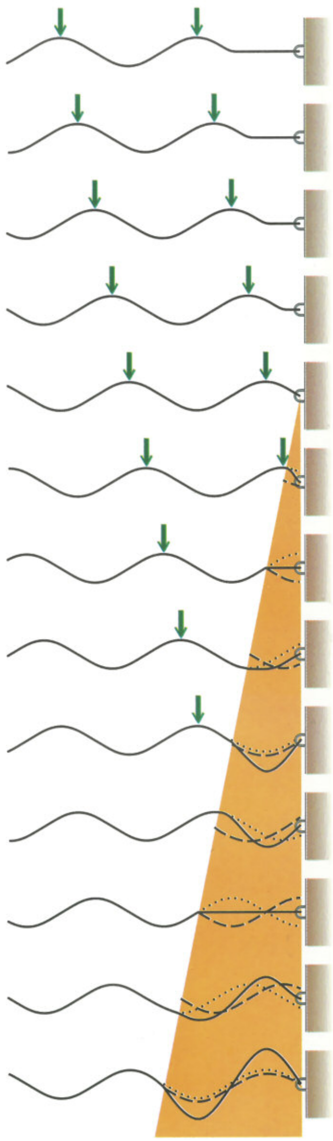


Fig. 16-35 A wave traveling to the right is reflected, and a standing wave begins to be formed at the right end of the string, indicated by shading. The string's displacement in this shaded region is a superposition of the original wave traveling to the right, *dotted line,* and a reflected wave traveling to the left, *dashed line.*

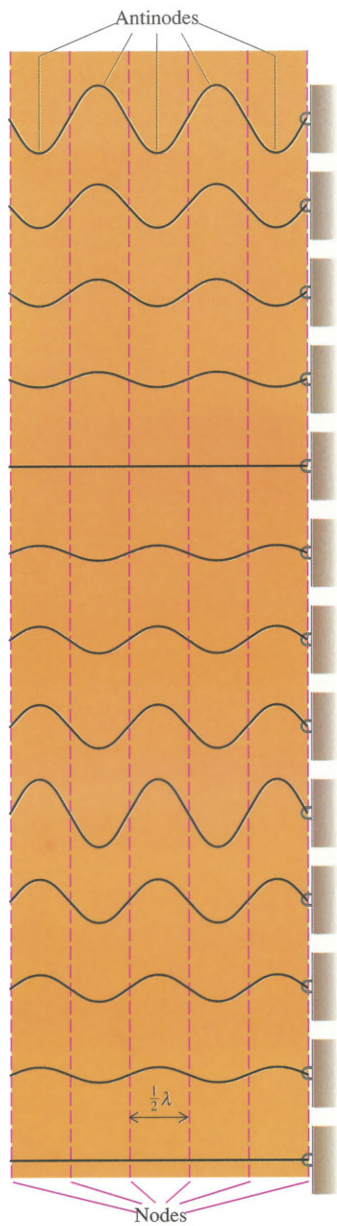


Fig. 16-36 The standing wave now extends over a larger region, indicated by the shading.

As the reflected wave travels farther to the left, the region of wave superposition grows (Fig. 16-36). At some points in the superposition region, the displacement is at times greater than the amplitude of the incident wave; at other points, *the displacement is always zero.* The points of **zero displacement** are called **nodes,** and the points of **maximum displacement** are called **antinodes.** The location of the nodes and antinodes is fixed, and the wave is therefore called a **standing wave.** The distance between adjacent nodes is $\frac{1}{2}$ wavelength, as indicated in Fig. 16-36:

$$x = \frac{\lambda}{2} \quad \text{(distance between adjacent nodes)} \quad (16\text{-}25)$$

This is also the distance between adjacent antinodes, which are midway between the nodes.

The fact that standing waves result from the superposition of waves traveling in opposite directions can be shown algebraically. This is accomplished when we add the expressions $y_1 = A \sin\left[2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)\right]$ and $y_2 = A \sin\left[2\pi\left(\frac{t}{T} + \frac{x}{\lambda}\right)\right]$, representing the displacements of the incident and reflected waves, respectively. See Problem 64.

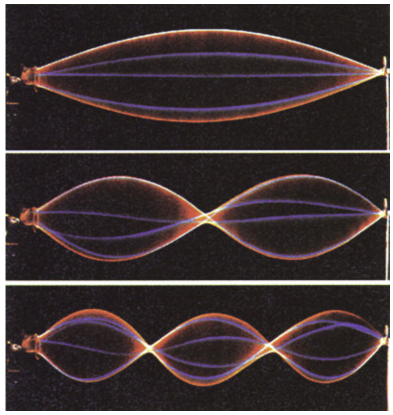


Fig. 16-37 A standing-wave pattern is formed when the length of the string is an integral number of half wavelengths; otherwise no standing wave is formed.

Eventually the standing wave reaches the left end of the string (in the time it takes for the reflected wave to travel from the right end). When this happens, the wave is again reflected—this time to the right. If this second reflected wave happens to be in phase with the original wave, the standing wave pattern is reinforced—its amplitude increases (Fig. 16-37a). If the second reflected wave is not in phase with the original wave, the standing-wave pattern is destroyed. The ends of the string are fixed.* So, to have a standing wave, the ends of the string must be at nodes of the standing wave. Since there is a node at every half wavelength, the string's length ℓ must be equal to an integral number n of half wavelengths:

$$\ell = \frac{n\lambda}{2}$$

This relationship between ℓ and λ may be stated as a condition on the values of the wavelength of standing waves that can be produced on a string of length ℓ. Solving the equation above for λ, we find

$$\lambda = \frac{2\ell}{n} \quad (\text{for } n = 1, 2, 3, \ldots) \quad (16\text{-}26)$$

that is,

$$\lambda = 2\ell, \ell, \frac{2\ell}{3}, \ldots$$

In order for a standing wave to be produced on a string of length ℓ, the oscillator must have a frequency f_n, found by inserting Eq. 16-26 into the equation $f = \frac{v}{\lambda}$:

$$f_n = n\frac{v}{2\ell} \quad (\text{for } n = 1, 2, 3, \ldots) \quad (16\text{-}27)$$

*The end of the string to which the oscillator is attached is not exactly fixed but oscillates with an amplitude that is small compared with the oscillation amplitude at an antinode. So the oscillation end of the string is close to being a node of the standing wave.

The oscillator is the source of energy. In an ideal system the amplitude would increase without limit as the waves are reflected at each end of the string. In practice, the amplitude does increase to the extent that the amplitude of oscillation at the antinodes is much greater than the amplitude of the oscillator. Hence the oscillator is close to a node of the system, and the oscillator end of the string can be considered a fixed end. If the oscillator is stopped, ideally the standing wave would continue indefinitely. In actuality, the wave quickly dies out. The energy delivered by the oscillator is necessary to compensate the energy loss during the reflection at the ends and the energy loss caused by air resistance.

If the frequency of the oscillator is not one of the standing-wave frequencies, the reflected waves will be out of phase at each reflection, and there will be destructive interference as often as constructive interference. The resulting wave will have an irregular shape and a small amplitude.

Resonant Frequencies; Harmonics

In Chapter 15 we described the phenomenon of resonance and introduced the concept of resonant frequency. When a body is forced to oscillate at one of its resonant frequencies, its amplitude of oscillation is large. The resonant frequencies of a vibrating string of length ℓ are the standing-wave frequencies given by Eq. 16-27 $\left(f_n = n\frac{v}{2\ell}, \text{ where } n = 1, 2, 3, \ldots\right)$. The lowest resonant frequency, called either the "fundamental frequency" or the "first harmonic," corresponds to setting $n = 1$ in this equation:

$$f_1 = \frac{v}{2\ell} \quad \textbf{(fundamental frequency, or first harmonic)} \tag{16-28}$$

The frequency corresponding to $n = 2$ in Eq. 16-27 has twice the value of the first harmonic and is called the second harmonic:

$$f_2 = 2f_1 \quad \textbf{(second harmonic)}$$

The third harmonic ($n = 3$) equals three times the fundamental f_1:

$$f_3 = 3f_1 \quad \textbf{(third harmonic)}$$

The nth harmonic equals n times the fundamental frequency:

$$f_n = nf_1 \quad \textbf{(}\boldsymbol{n}\textbf{th harmonic)} \tag{16-29}$$

Fig. 16-38 shows the first three harmonics of a stretched string.

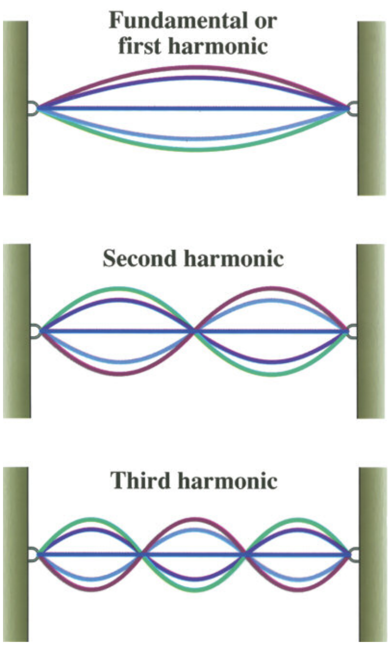


Fig. 16-38 Harmonics of a vibrating string.

EXAMPLE 12 The First Three Harmonics of a Vibrating String

A string of mass 2.00 g and length 1.00 m is fixed at one end and attached at the other end to an oscillator of variable frequency. The string is under a tension of 51.0 N. Find the three lowest oscillator frequencies for which standing waves will be formed.

SOLUTION Waves propagate along the string at a speed that may be found by applying Eq. 16-3:

$$v = \sqrt{\frac{F}{\mu}}$$

The mass density μ is 2.00 g/m, or 2.00×10^{-3} kg/m. Thus

$$v = \sqrt{\frac{51.0\ \text{N}}{2.00 \times 10^{-3}\ \text{kg/m}}} = 160\ \text{m/s}$$

Applying Eq. 16-28, we find the minimum oscillator frequency f_1 for the formation of a standing wave:

$$f_1 = \frac{v}{2\ell} = \frac{160\ \text{m/s}}{2(1.00\ \text{m})} = 80.0\ \text{Hz}$$

This is the lowest resonant frequency of the string, that is, its fundamental frequency, or first harmonic. The corresponding wavelength $\lambda_1 = \frac{v}{f_1} = \frac{160\ \text{m/s}}{80.0\ \text{Hz}} = 2.00$ m. This wavelength equals twice the length of the string ($\lambda = 2\ell$), as illustrated in the sketch of first harmonic vibrations in Fig. 16-38.

The two next-lowest standing-wave, or resonant, frequencies are the second harmonic f_2 (two times the fundamental) and the third harmonic f_3 (three times the fundamental):

$$f_2 = 2f_1 = 2(80.0\ \text{Hz}) = 160\ \text{Hz}$$

$$f_3 = 3f_1 = 3(80.0\ \text{Hz}) = 240\ \text{Hz}$$

Corresponding to the second harmonic is the wavelength $\lambda_2 = \frac{v}{f_2} = \frac{160\ \text{m/s}}{160\ \text{Hz}} = 1.00$ m, which equals the length of the string. Corresponding to the third harmonic is the wavelength $\lambda_3 = \frac{v}{f_3} = \frac{160\ \text{m/s}}{240\ \text{Hz}} = 0.667$ m, two thirds the length of the string. See Fig. 16-38.

Harmonic Analysis

Consider a wave displacement y that is a sum of two displacements y_1 and y_2, each representing a different resonant frequency of a string fixed at both ends. According to the superposition principle, y represents a possible wave on the string. For example, it is possible for the string to vibrate simultaneously at both its first and second harmonics, as illustrated in Fig. 16-39.

Conversely, one can consider any vibration of the string to be a superposition of its resonant frequencies. For example, if a stretched string is plucked at its center, the wave motion that results can be resolved into a sum of harmonics, each having its own amplitude.

A vibrating string will cause the surrounding air to vibrate at the same frequency, producing a sound wave. (If the string is part of a musical instrument, an air cavity, the "sound box," in the instrument enhances and amplifies the sound, forming standing sound waves, which we shall examine next.)

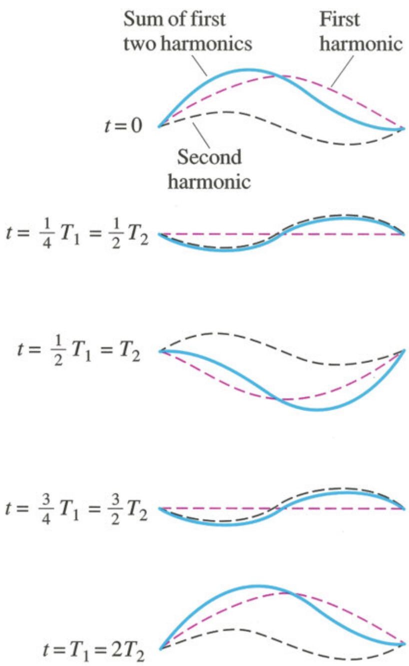


Fig. 16-39 A string vibrates at both its first and second harmonics.

EXAMPLE 13 The Right Tension for a Guitar's Thickest String

The thickest string on a guitar has a mass per unit length of 5.60×10^{-3} kg/m (Fig. 16-40). The string is stretched along the neck of the guitar and is free to vibrate between two fixed points 0.660 m apart. When plucked, this string vibrates at a fundamental frequency of 165 Hz and produces a sound wave of the same frequency, which corresponds to the musical note E below middle C. In order for the string to have this fundamental frequency, the tension in it must be adjusted to the right value. Find that value.

Fig. 16-40

SOLUTION We apply Eq. 16-28 $\left(f_1 = \frac{v}{2\ell}\right)$ for the fundamental frequency and Eq. 16-3 $\left(v = \sqrt{\frac{F}{\mu}}\right)$ for the speed of the wave on a string of mass per unit length μ, under tension F.

$$f_1 = \frac{v}{2\ell} = \frac{1}{2\ell}\sqrt{\frac{F}{\mu}}$$

Solving for F, we find

$$F = (2\ell f_1)^2\mu = [2(0.660\text{ m})(165\text{ Hz})]^2(5.60 \times 10^{-3}\text{ kg/m})$$
$$= 266\text{ N}$$

The same kind of analysis we have applied to a vibrating string can be applied to any other vibrating body. The vibrations can always be resolved into a linear sum of vibrations at certain resonant frequencies. The values of the resonant frequencies will depend on the particular body considered and will not in general be a simple multiple of the fundamental frequency. Consider, for example, the vibration of a drum. When struck, the drum's flexible membrane vibrates at various resonant frequencies. The fundamental frequency f_1 depends on the size of the drum—the larger the drum, the lower the fundamental frequency is. The next few resonant frequencies are approximately $1.59f_1$, $2.14f_1$, $2.30f_1$, $2.65f_1$. Because these are not integral multiples of f_1, they are not harmonics. This fact is responsible for the nonmelodious nature of the sound produced by a drum, in contrast to that produced by string or wind instruments. These latter instruments have resonant frequencies that are integral multiples of their fundamental frequencies, and so the sounds produced are pleasing to the ear, hence the name "harmonics."

A tuning fork is a device used to tune musical instruments. It is designed so that, when struck, it vibrates at essentially only one frequency; in other words, the amplitude of vibration of its fundamental frequency is much greater than the amplitude of any of its other resonant frequencies. The size of the tuning fork determines its frequency.

Standing Sound Waves

Suppose that a vibrating tuning fork is placed near the left end of a pipe that is open at both ends. A sound wave is generated, and sound can be heard at the right end of the pipe. The amplitude and loudness of the sound will depend on the frequency of the tuning fork and on the length of the pipe. The air in the pipe has certain resonant frequencies. If the tuning fork frequency is close to one of the pipe's resonant frequencies, the sound heard at the right end will have a relatively large amplitude. This situation is analogous to the formation of a standing wave on a string. The sound wave that originates at the left end of the pipe travels to the other end and is partially reflected; the part of the wave energy not reflected is transmitted out through the opening. The reflected sound wave then travels back to the left and is again partly reflected. This process is repeated. Unless the various reflected waves are all in phase, there is much destructive interference and only a small-amplitude wave results. This will be the case unless the frequency of the tuning fork is at a resonant frequency of the pipe. If the tuning fork is at a resonant frequency, the reflected waves are all in phase and a standing wave is formed.

Each of the pipe's open ends is a displacement antinode* since the air is free to move there. A standing wave will be formed only if the wavelength of the sound is related to the length of the pipe in such a way that antinodes are located at the ends (Fig. 16-41). Since the antinodes are a half wavelength apart, the pipe's length ℓ must be an integral number n of half wavelengths:

$$\ell = n\frac{\lambda}{2} \qquad \text{(for } n = 1, 2, 3, \ldots\text{)}$$

or

$$\lambda = \frac{2\ell}{n}$$

The resonant frequencies of the pipe are found when these values of λ are inserted into the equation $f = \dfrac{v}{\lambda}$:

$$f_n = n\frac{v}{2\ell} \qquad \text{(for } n = 1, 2, 3, \ldots\text{)} \qquad (16\text{-}30)$$

The lowest resonant frequency, that is, the **fundamental frequency,** or **first harmonic,** corresponds to setting $n = 1$ in the preceding equation:

$$f_1 = \frac{v}{2\ell} \qquad \text{(first harmonic of an open-ended pipe)} \qquad (16\text{-}31)$$

Second-, third-, and higher-order harmonics correspond to setting $n = 2$, $n = 3$, and so on in Eq. 16-30. These harmonics are multiples of the first harmonic:

$$f_2 = 2f_1$$

$$f_3 = 3f_1$$

$$\vdots$$

$$f_n = nf_1 \qquad \text{(nth harmonic of an open-ended pipe, } n = 1, 2, 3, \ldots\text{)} \qquad (16\text{-}32)$$

The first three harmonics of an open-ended pipe are shown in Fig. 16-41.

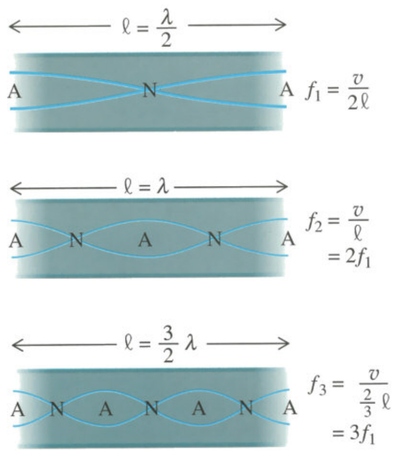


Fig. 16-41 The first three harmonics of sound in a pipe with both ends open.

*The antinode does not occur exactly at the pipe's end. But if the pipe's diameter is small compared with the wavelength of the sound (as it is in most musical instruments), an antinode is located close to an open end.

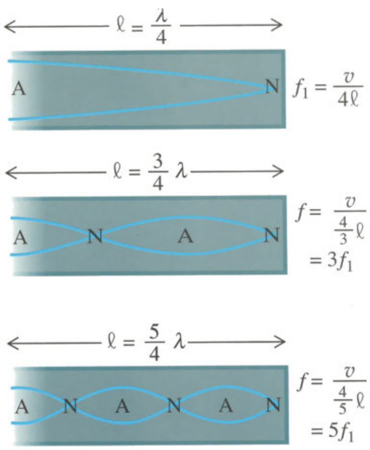


Fig. 16-42 The lowest three harmonics of sound in a pipe with one end closed. Notice that only odd harmonics are present.

A standing wave can also be formed in a pipe that is closed at one end. The closed end is a displacement node, since the air is not free to move there. The open end is a displacement antinode. For a standing wave to be produced, the sound's wavelength must be such that an antinode is formed at one end and a node at the opposite end. The longest wavelength that can satisfy this condition is one for which the pipe's length ℓ equals $\lambda/4$, or $\lambda = 4\ell$ (Fig. 16-42). The corresponding frequency, the pipe's lowest resonant frequency, the fundamental frequency, or first harmonic, is

$$f_1 = \frac{v}{\lambda}$$

or

$$f_1 = \frac{v}{4\ell} \quad \text{(first harmonic of a pipe closed at one end)} \tag{16-33}$$

The second longest wave that can satisfy the condition of a node at one end and an antinode at the other end corresponds to $\ell = \frac{3}{4}\lambda$, or $\lambda = \frac{4}{3}\ell$ (Fig. 16-42). The corresponding frequency $\dfrac{v}{\lambda} = \dfrac{3}{4}\dfrac{v}{\ell}$, which is three times the fundamental frequency:

$$f_3 = 3f_1$$

The third longest wave corresponds to $\ell = \frac{4}{5}\lambda$, and a frequency $\dfrac{v}{\lambda} = \dfrac{5}{4}\dfrac{v}{\ell}$, which is five times the fundamental frequency f_1:

$$f_5 = 5f_1$$

In general, we have as our resonant frequencies all odd multiples of the fundamental frequency f_1:

$$f_n = nf_1 \quad \text{(}n\text{th harmonic of a pipe closed at one end, } n = 1, 3, 5, \ldots\text{)} \tag{16-34}$$

No even harmonics are present for a pipe closed at one end.

Standing waves can also be produced in a pipe when a turbulent airflow is directed near one end (Fig. 16-43). This method is utilized in musical wind instruments, such as the organ and flute. In this case, the various resonant frequencies are produced simultaneously.

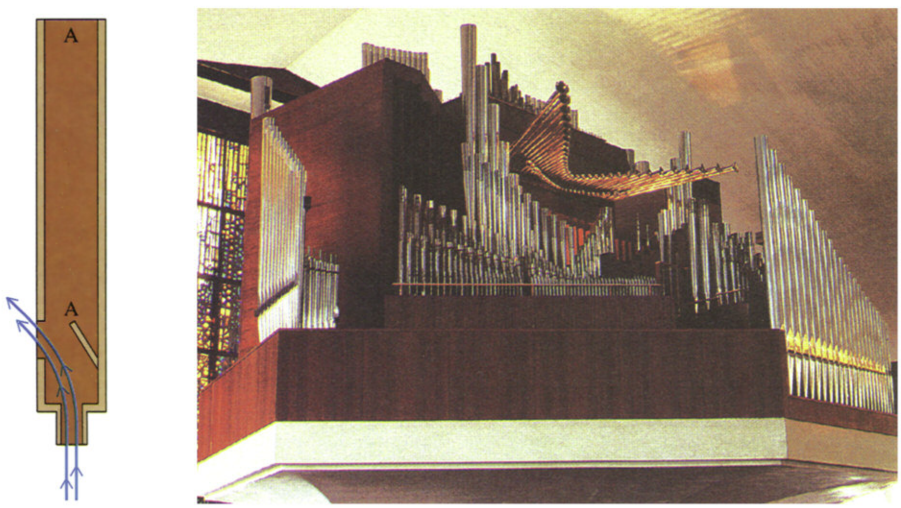


Fig. 16-43 A standing wave is produced by a turbulent flow of air through one end of a wind instrument such as an organ pipe.

EXAMPLE 14 Adjusting the Length of a Column of Air to Produce a Standing Wave

A tube is completely filled with water, and a tuning fork vibrating at 512 Hz is placed above it. The level of water in the tube is gradually reduced as water is drained from the bottom until a condition of resonance is reached, at which point the sound is loudest. Find the length ℓ of the air-filled cavity (Fig. 16-44).

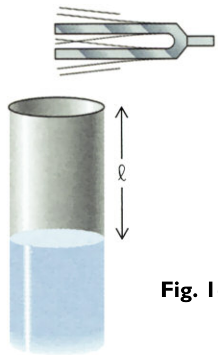


Fig. 16-44

SOLUTION What we are interested in here is the empty part of the tube. As we drain water out, we are in essence changing the length ℓ of a pipe closed at one end (the top of the water column causes the "pipe" to be closed at one end). Resonance occurs when the frequency of the tuning fork equals a resonant frequency of this "pipe" of length ℓ. Using the speed of sound at 20.0° C (344 m/s; Table 16-1), the wavelength of the sound produced by the tuning fork is:

$$\lambda = \frac{v}{f} = \frac{344 \text{ m/s}}{512 \text{ Hz}} = 0.672 \text{ m} = 67.2 \text{ cm}$$

From Fig. 16-41, we see that the shortest value of ℓ for resonance is $\frac{1}{4}$ wavelength. Thus $\ell = \frac{1}{4}\lambda = 16.8$ cm.

EXAMPLE 15 The Frequency of Sound From a Flute

All the holes of a flute of length 65.6 cm are covered, and a musical note is produced when the flute is blown into. The flute acts as a pipe open at both ends. Find the frequency of the sound produced.

SOLUTION We apply Eq. 16-31 to find the fundamental frequency of the flute, using the speed of sound at 20.0° C (344 m/s; Table 16-1):

$$f_1 = \frac{v}{2\ell} = \frac{344 \text{ m/s}}{2(0.656 \text{ m})} = 262 \text{ Hz}$$

This is the frequency of the musical note middle C.

A string instrument, such as a guitar, violin, or piano, utilizes an air cavity to amplify the sound produced. Each instrument, of course, has its own characteristic sound. Two instruments can produce the same note and yet sound quite different from each other. The musical note is determined by the fundamental frequency of the sound; the differences are in the harmonic structure of the sounds. Different instruments produce different relative amplitudes of the harmonics when the same musical note is played. The human ear detects the harmonic structure of the sounds and identifies them with the respective instruments. See Fig. 16-45. This harmonic structure is determined by the size and shape of the air cavity, the structure of the sounding board, the characteristics of the string, and the interaction of these three factors, together with where and how the string is sounded—whether it is plucked, bowed, or hammered.

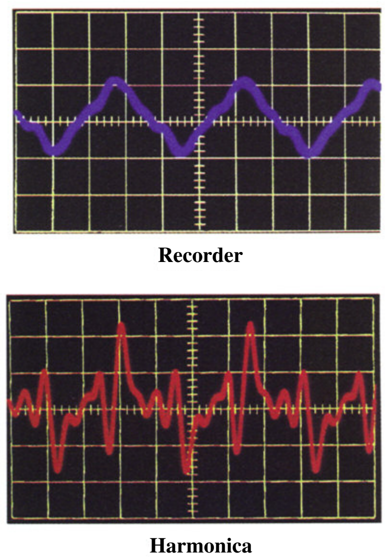


Fig. 16-45 The same musical note A, $f_1 = 440$ Hz, is produced on two instruments. The sounds produced are quite different because of their different harmonic structures.

The Ear

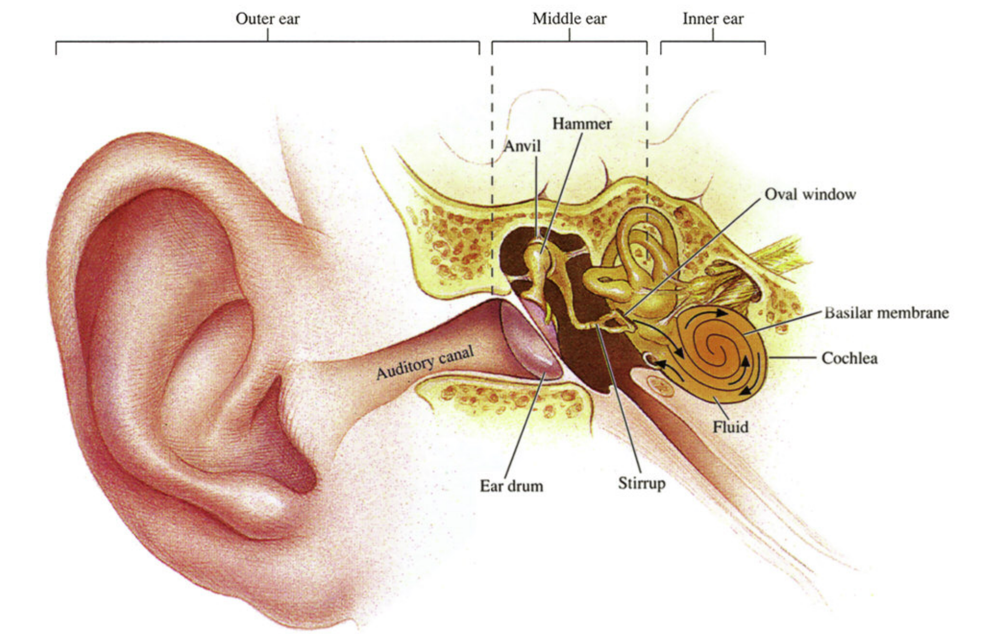


Fig. 16-A Simplified diagram of the ear.

The human ear is a remarkably sensitive detector of sounds. It is able to detect faint sounds with displacements smaller than an atomic diameter, as seen in Example 9. Furthermore, the ear is capable of harmonic analysis of the sounds it hears. For example, you can immediately recognize the difference between a guitar and a piano playing the same musical note. Somehow your ear senses the harmonic components of the two sounds, and, utilizing your memory of the harmonics of each instrument, you identify one source as a guitar and the other as a piano. Similarly, you are usually able to recognize familiar voices over the telephone. The sound waves transmitted by the telephone receiver are completely determined by their harmonic components. Thus recognition of voices consists of some kind of harmonic analysis by the ear and brain.

The 3 cm long auditory canal in the outer ear is essentially an air-filled pipe closed at one end. Its fundamental frequency corresponds to a wavelength $\lambda = 4\ell = 4(3 \text{ cm}) = 12$ cm, or a frequency $f = \dfrac{v}{\lambda} = \dfrac{340 \text{ m/s}}{0.12 \text{ m}} = 2800$ Hz. The maximum sensitivity of human hearing occurs at about this frequency (roughly that of a baby's cry). Thus the length of the auditory canal explains why the ear should be particularly sensitive to frequencies close to 2800 Hz. However, this factor alone cannot explain the broad range of frequencies to which the ear can be highly sensitive (roughly 20 Hz to 20,000 Hz for 1% of the population). The most important frequency-dependent effects occur in the cochlea of the inner ear, shown in Fig. 16-A.

Connecting the cochlea's oval window to the eardrum at the end of the auditory canal are the three bones of the middle ear (hammer, anvil, and stirrup). These bones serve as a system of levers that roughly doubles the force of vibrations transmitted from the eardrum to the oval window. Since the area of the oval window is much less than the area of the eardrum, the pressure of sound waves is greatly increased as the wave passes into the viscous fluid within the cochlea. Sound waves propagate through that fluid from the oval window to the tip of the cochlea, returning along the other side of the basilar membrane, as indicated by arrows in the figure. Pressure differences between the fluids on either side of the basilar membrane displace the membrane laterally. Tens of thousands of nerve endings along the membrane sense this displacement. These nerve endings then transmit electrical pulses to the brain, and we hear. Information about the harmonics present in the sound wave are provided by the location of the stimulated nerve endings along the basilar membrane and by the rate of transmission of pulses.

CHAPTER 16 SUMMARY

Mechanical waves, such as sound, water waves, and waves on a string, require a material medium through which the wave travels. Electromagnetic waves, including light, require no medium; they can travel through a vacuum.

For a continuous, periodic wave (as opposed to a wave pulse) of frequency f, the motion of the medium at each point in the wave is a periodic disturbance that has the same frequency f. This frequency is determined by the wave source.

The distance between two successive identical points on a wave is called the "wavelength," denoted by λ. The speed v at which the wave energy and wave form move is determined by the medium. The speed, wavelength, and frequency of any wave are related by the equation

$$v = \lambda f$$

A wave on a string of mass per unit length μ, under a tension F, travels at a speed

$$v = \sqrt{\frac{F}{\mu}}$$

Sound travels through a diatomic, ideal gas at a speed

$$v = \sqrt{\frac{1.40kT}{m}}$$

where k is Boltzmann's constant, T is absolute temperature, and m is the molecular mass.

Water waves having wavelengths that are greater than 10 cm but much less than the depth of the water travel at a speed

$$v = \sqrt{\frac{g\lambda}{2\pi}}$$

The Doppler effect is the phenomenon that occurs when either the source of a wave or an observer moves relative to the medium through which the wave travels. The result is that the observed frequency f_O is different from the source frequency f_S. For mechanical waves, the two frequencies are related by the equation

$$f_O = f_S\left(\frac{1 \pm v_O/v}{1 \mp v_S/v}\right) \quad \text{(upper signs if toward; lower signs if away)}$$

where the upper sign in both numerator and denominator applies when the motion of either the observer or the source is toward the other and the lower sign applies when either the observer or the source moves away from the other.

The power transmitted by a harmonic wave on a string is

$$P = 2\pi^2\mu v f^2 A^2$$

where μ is the string's mass per unit length, v is the wave speed, f is the frequency, and A is the amplitude. The intensity of a harmonic sound wave is given by a similar expression (with the mass density ρ replacing μ):

$$I = 2\pi^2\rho v f^2 A^2$$

where intensity is defined as the power per cross-sectional area $\mathscr{A}$:

$$I = \frac{P}{\mathscr{A}}$$

The intensity level β of a sound wave, measured in decibels, is defined as

$$\beta = 10 \log\left(\frac{I}{I_0}\right)$$

where the reference level $I_0 = 10^{-12}$ W/m^2 is the lowest sound intensity perceptible by individuals with normal hearing.

The wave displacement y in a harmonic wave traveling in the positive x direction is expressed in terms of position x and time t as

$$y = A \sin\left[2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)\right]$$

where A is the amplitude of the wave, T is the period, and λ is the wavelength. For a wave traveling in the negative x direction, the corresponding expression is

$$y = A \sin\left[2\pi\left(\frac{t}{T} + \frac{x}{\lambda}\right)\right]$$

The superposition principle states that when two or more wave disturbances are present in a medium the displacement at any point is the sum of the individual displacements.

When two sources produce waves that either are in phase or have a constant phase difference, the sources are said to be coherent. Coherent wave sources can produce observable interference effects at various points in space.

When the waves at a given point are in phase, they interfere constructively, and the resulting amplitude is maximum—the sum of the two amplitudes. When waves are 180° out of phase, they interfere destructively, and the amplitude is the difference of the two amplitudes—equal to zero if the amplitudes are equal.

In general, the phase difference $\Delta\phi$ between two waves, arising from a path-length difference Δx, is

$$\Delta\phi = 2\pi\,\frac{\Delta x}{\lambda}$$

Constructive interference occurs when

$$\Delta x = 0, \lambda, 2\lambda, 3\lambda, \ldots$$

Destructive interference occurs when

$$\Delta x = \frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2}, \ldots$$

Two waves of slightly different frequencies f_1 and f_2 produce beats with a beat frequency $f_B = f_1 - f_2$.

Standing waves are waves in which, at certain points, called "nodes," the displacement is always zero, whereas at other points, called "antinodes," the displacement is maximum. Standing waves are formed when waves are reflected from the boundaries of the wave medium. The distance Δx between adjacent nodes in any standing wave equals $\frac{1}{2}$ wavelength:

$$\Delta x = \frac{\lambda}{2}$$

For a standing wave on a string having both ends fixed, the two ends are located at nodes of the standing wave, and so the length is a multiple of $\frac{1}{2}$ wavelength:

$$\ell = \frac{n\lambda}{2}$$

or

$$\lambda = \frac{2\ell}{n} \qquad (\text{for } n = 1, 2, 3, \ldots)$$

A wave having such a wavelength will be produced if the frequency of the source is appropriate for the wave's speed v and length ℓ:

$$f = \frac{v}{\lambda}$$

with λ given above, or

$$f_n = n\frac{v}{2\ell} \qquad (\text{for } n = 1, 2, 3, \ldots)$$

These frequencies are the resonant frequencies of the string. The lowest resonant frequency f_1 is called the "fundamental frequency," or the "first harmonic," f_2 is called the "second harmonic," f_3 is the "third harmonic," and so forth.

For a standing sound wave in a pipe with both ends open, each end is an antinode, and the harmonic frequencies are the same as for a string:

$$f_n = n\frac{v}{2\ell} \qquad (\text{for } n = 1, 2, 3, \ldots)$$

In a pipe open at one end and closed at the other, there is a node at the closed end and an antinode at the open end; the harmonics, which are lower than for an open pipe of the same length, are given by

$$f_n = n\frac{v}{4\ell} \qquad (\text{for } n = 1, 3, 5, \ldots, \text{ odd only})$$

Any standing wave can be regarded as a superposition of various harmonics.

Questions

1 Suppose you throw someone a ball, thereby transporting kinetic energy. Could you consider the motion of the ball to be a mechanical wave pulse?

2 Which quantities determine the speed of a particle in a medium through which a wave propagates: (a) wave speed; (b) wavelength and frequency; (c) amplitude and frequency; (d) wave speed and period?

3 Would it be possible to detect on earth sounds produced on another planet if you had a detector sensitive enough to very low-intensity sounds?

4 Suppose you were standing on the moon as a nearby lunar landing module approached the surface, firing its engines to slow its descent. Could you hear the engine?

5 When cars begin to move in a long line of stalled traffic, the motion passes through the line as a wave pulse.
(a) What is the direction of motion of the pulse relative to the motion of the cars?
(b) Is the wave speed affected by the drivers' reaction times?

6 Creating a "wave" is a popular pastime at football games and other sporting events. The wave begins when everyone in one section of the stadium quickly stands up and then sits down. Then people sitting in an adjacent section, say, to the right, respond with the same motion but delayed a bit. Next the section to their right follows. The result is a wave pulse through the spectators. Does human reaction time affect the wave's (a) amplitude; (b) period; (c) wave speed; (d) wavelength?

7 When lightning strikes at some distance, will you see the lightning first or hear the accompanying thunder first? Explain.

8 Suppose you are in a rowboat on a quiet lake as a fast motorboat passes at a considerable distance. The motorboat produces waves, which are a superposition of harmonic waves of various wavelengths. Do the waves of longer or shorter wavelengths reach you first?

9 A source of sound and an observer both move through the air along the same line at the same velocity so that there is zero velocity of the source relative to the observer.

(a) Will the observer hear a Doppler-shifted frequency?

(b) Will the time it takes for the sound to travel from the source to the observer be different from what it would be if both were at rest?

10 Large supersonic transport planes (SST's) produce a sonic boom, which many people find very objectionable. Suppose an SST is to travel coast to coast. Would it be reasonable for the plane to head out over the ocean immediately after takeoff, break the sound barrier there and then head back over land to avoid creating a sonic boom over land?

11 By what factor must the amplitude of a sound wave be increased in order to increase the intensity level by 10 dB?

12 If a 100 Hz harmonic sound wave produces a 60 dB sound at a given point, what is the intensity level of a 1000 Hz harmonic sound wave having the same amplitude at the same point?

13 If a jet engine produces at takeoff a 100 dB sound 100 m from the aircraft, at what distance should the sound's intensity level be 80 dB? Assume that the sound is not reflected or absorbed by the surroundings.

14 Some of the strings on a classical guitar are single strands of plastic of varying diameters, whereas others are plastic wrapped with wire. Thus the mass of the strings varies considerably. Will the sound produced by the heaviest string have a higher or lower frequency than the sound produced by the other strings?

15 A guitar is tuned when the tension in its strings is adjusted. Should you increase or decrease the tension in a string to produce a higher-frequency sound?

16 Will standing sound waves have a lower fundamental frequency in pipe A, which is open at both ends, or in pipe B, of equal length, open at one end and closed at the other?

17 Suppose you replace the air in a pipe by helium.

(a) How would this affect the resonant frequencies of sound waves in the pipe?

(b) Suppose you inhale some helium from a helium-filled balloon. What happens to your voice? Explain.

Answers to Odd-Numbered Questions

1 no; **3** no; **5** (a) opposite; (b) yes; **7** see it first; **9** (a) no; (b) yes; **11** $\sqrt{10}$; **13** 1 km; **15** increase; **17** (a) increases them; (b) much higher pitch

Problems (listed by section)

16-1 Description of Waves

1 A cork floating on a lake bobs up and down as a small wave passes. The cork completes 4.00 cycles in 1.00 s. The wave peaks are 10.0 cm apart. Find the speed of the wave.

2 Tsunamis (or tidal waves) are very-long-wavelength water waves generated by earthquakes. A tsunami originating in Japan has a wavelength of 10^5 m and a period of 10 minutes. How long does it take to travel across the Pacific Ocean to California, 8000 km away?

3 Radio waves are electromagnetic waves that travel at a speed of 3.00×10^8 m/s, the speed of light. An AM radio station has an assigned frequency of 890 kHz, which means that the radio waves broadcast by the station are at this frequency. Find the wavelength of these radio waves.

4 Suppose you are listening to a song on your radio. You change to another station playing the same song. If the second station broadcasts at a higher radio frequency than the first, will that affect the wavelength of the sound you hear?

5 You see lightning strike a mountaintop, and 3.00 s later you hear the accompanying thunder. How far away is the mountaintop? The speed of sound is 340 m/s, and the speed of light is 3.00×10^8 m/s.

★ **6** Earthquakes generate two types of waves, which travel through the earth: (1) *primary,* or P, waves are longitudinal *pressure* waves that have the greater speed (about 5 km/s in the crust) and are therefore the first to be felt at some distance from the center of the earthquake, and (2) *secondary,* or S, waves are transverse, *shear* waves that are somewhat slower ($v \approx 3$ km/s). The location of the earthquake can be determined by recording the arrival times for these waves on seismographs at various locations. Suppose an S wave is recorded 2 min after a P wave. About how far away from the seismograph is the earthquake's center?

★ **7** Suppose that there is a seismic P wave traveling at 5.0×10^3 m/s with a wavelength of 2.0×10^3 m.
(a) Find the wave's frequency.
(b) Find the average speed of a particle of the earth's surface at a point where the wave amplitude is 2.0 cm.

★ **8** When a transverse wave of amplitude 2.00 cm and wavelength 20.0 cm passes through a medium, the average speed of a particle of the medium is 4.00 m/s. Find the speed of the wave.

9 A nerve impulse is a wave pulse that travels along a nerve, typically at a speed of 50 m/s. If the pulse sweeps past one point in the nerve from $t = 0$ to $t = 2$ ms, during what time interval will it pass a point in the nerve 1 m away?

16-2 Wave Speed

10 By what percent must one increase the tension in a guitar string to change the speed of waves on the string from 300 m/s to 330 m/s?

11 A 1.00 m long string of mass 0.0100 kg, under a tension of 100 N, transmits a wave of amplitude 2.00 cm and wavelength 10.0 cm.
(a) Find the speed of the wave form.
(b) How far does a particle of the string travel during one cycle?
(c) What is the average speed of a particle of the string?

★ **12** A 2.00 m long string of mass 10.0 g is attached to a 3.00 m long string of mass 30.0 g (Fig. 16-46). The strings are under a tension of 100 N.
(a) How long will it take for a wave pulse to travel from point A to point C?
(b) How long will it take for reflected pulses to return to point A? Indicate the orientation of the reflected pulses relative to the original pulse.

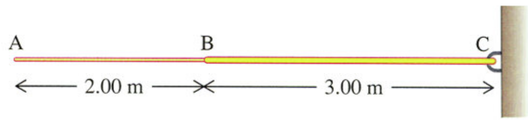


Fig. 16-46

13 The normal human ear is sensitive to sounds with frequencies from about 20 Hz to about 20,000 Hz. What is the corresponding range of wavelengths (a) in air; (b) in water?

14 If you hear your echo 3.00 s after you shout, how far away is the mountain reflecting the sound?

15 Suppose you are in your dorm listening on your radio to a football game being played on campus, 500 m away. As a touchdown is scored, you hear on the radio the roar of the crowd. When do you hear the crowd's cheers directly from the stadium?

16 Find the frequency of a deep-water wave of wavelength 30.0 cm.

17 You are floating in the ocean in deep water as a 5.00 m wavelength wave passes. How long do you wait between wave peaks?

18 Find the speed of deep-water waves having a frequency of 1.00 Hz.

★ **19** A fisherman sits in a stationary boat in the middle of a lake as a high-powered motorboat passes at some distance. The fisherman notices that the wavelengths of the waves from the wake gradually decrease, with 10.0 cm waves arriving 60.0 s after 30.0 cm waves. How far away was the motorboat?

★ **20** A boat floating at rest encounters an unusually big deep-water ocean wave with an amplitude of 15.0 m and a wavelength of 200 m. Find the boat's speed and the magnitude of its acceleration.

21 Sonar depth finders are used on boats to determine the depth of water by reflecting a pulse of sound from the bottom. What is the depth of water if there is a 0.10 s delay between emission of the pulse and detection of the reflected pulse?

22 At what temperature would sound travel through air at 400 m/s?

23 How much longer would it take for sound to travel 1.0 km through arctic air at $-50°$ C than to travel 1.0 km through desert air at $+50°$ C?

24 A bat emits ultrasonic pulses and uses them to navigate and to locate flying insects. If these pulses are sent at a rate of 4 per second, what is the maximum distance a reflecting object can be if the reflected pulse is to be received by the bat before the next pulse is emitted?

25 What is the wavelength of a 5.00×10^4 Hz sound wave pulse emitted by a bat?

16-3 The Doppler Effect

★ 26 Suppose you are in a high-speed boat moving at 20.0 m/s directly into approaching 1.00 m wavelength waves. What is the time interval between wave peaks hitting the boat?

27 A surfer rides a wave, moving at the same speed as the wave. What is the frequency of the wave, as observed by the surfer?

★ 28 Suppose you want to demonstrate the Doppler effect for deep-water waves, using a 6.00 Hz source moving toward a stationary observer. How fast would the source have to move through the water if the frequency of the waves seen by the observer is to be twice the frequency of the source?

29 A sound source moves through air toward a stationary observer. The frequency of the sound the observer hears is 20.0% higher than the source frequency. How fast is the source moving?

30 Suppose that a 1024 Hz tuning fork moves at 10.0 m/s through (a) air and (b) water. For each medium, find the observed frequency of the sound at a point directly in front of the source.

★★ 31 The driver of a car hears the hum of its engine at a frequency of 200 Hz.

(a) Find the frequency of the sound heard by a pedestrian standing beside the road, first as the car approaches at a speed of 20.0 m/s and then after it passes.

(b) Now suppose that the wind is blowing at a speed of 20.0 m/s in the direction of the car's motion. What would be the frequencies of the sound heard by the pedestrian as the car approaches and passes?

★ 32 Bats use the Doppler effect as a directional guide and to detect insects. As it flies at a speed of 5.00 m/s, a bat emits a brief pulse of 60.0 kHz ultrasound in the forward direction. A nearly stationary insect in front of the bat reflects sound back to the bat. Find the frequency of the sound detected by the bat. (Bats are particularly sensitive to 61.8-kHz sound.)

★ 33 As you stand beside the German Autobahn (where there are no speed limits), a Porsche passes while sounding its horn. The frequency of the sound you hear drops from 500 Hz as the car approaches to 300 Hz after the car passes. How fast was it moving?

34 Sound from a foghorn in a lighthouse has a frequency of 100 Hz. Suppose that during a storm 40.0 m/s winds blow by the lighthouse. What is the frequency of sound heard by a stationary observer (a) downwind and (b) upwind from the lighthouse?

★ 35 A driver sounds his horn as he drives at 25.0 m/s toward a tunnel in the side of a mountain. The mountainside reflects the sound, and the driver hears an echo. Find the frequency of the echo heard by the driver if the frequency emitted by the horn is 500 Hz.

★★ 36 Suppose that, as you are driving, a stationary police radar unit in front of you detects a radar signal reflected from your car. The ratio of the reflected signal frequency to the emitted frequency differs from unity by 2×10^{-7}. How fast are you going?

★ 37 A police car moving at 20.0 m/s follows a speeding car moving at 30.0 m/s. A radar unit in the police car emits a 1.00×10^{10} Hz signal, which is reflected by the other car. What is the difference between this emitted frequency and the frequency of the reflected signal detected by the radar unit?

★ 38 A star is moving away from the earth at 50.0 km/s. By how much will the H_α line ($f = 4.571 \times 10^{14}$ Hz) be shifted and in what direction?

★ 39 A supersonic jet passes directly overhead at an altitude of 12,000 m, traveling at 680 m/s, or mach 2. How much time elapses before you hear the sonic boom?

16-4 Power and Intensity; the Decibel Scale

40 Two people talk simultaneously. If the intensity level is 60 dB when either one speaks alone, what is the intensity level when both speak at once?

41 One hundred people at a party are talking at once. On the average, each person alone produces 65 dB sound near the center of the room. Find the intensity level produced by all 100.

42 When a 100 dB sound wave comes through an open window of area 0.500 m^2, how much acoustic energy passes through the window in a 10.0 min interval?

43 At a football game, 100,000 spectators produce sound that is heard 1.00 km away, where the intensity level is 60.0 dB. Assuming that no sound is reflected or absorbed, we can treat the sound as radiating equally in all directions, so that the intensity is constant over a hemispherical surface. How much acoustic power is generated by the fans?

44 The driver of a car honks her horn as she enters a narrow tunnel. If the intensity level is 80.0 dB 20.0 m in front of the car, what is the intensity level 100 m in front of the car? Assume that no sound is absorbed by the tunnel walls.

45 Find the intensity level of a 50.0 Hz sound wave in which the amplitude of vibration is 1.00 mm.

46 Only half the population can hear a 60 dB, 60 Hz sound, but nearly everyone can hear a 100 dB, 60 Hz sound. What is the ratio of the amplitudes of these waves?

47 Find the intensity of a 53.0 dB sound.

48 What is the minimum power required for a loudspeaker to produce 105 dB sound at a distance of 10.0 m in any direction?

16-5 Time Dependence of the Displacement of a Particle of the Medium

49 A wave on a string is described by the equation $y = 5 \sin(4\pi t - 0.1\pi x)$, where x and y are in cm and t is in s. What are the values of the amplitude, frequency, wavelength, and speed of this wave? In what direction does it travel?

50 For the wave described in Problem 49, graph y versus x at $t = 0$, $t = 0.125$ s, and $t = 0.250$ s.

51 A distant 512 Hz tuning fork produces a plane sound wave that moves along the positive x-axis and has an amplitude of 1.00×10^{-8} m. Write an expression for the displacement as a function of x and t.

52 A wave is described by the equation $y = 3 \sin(2x + 10t)$, where x is in cm and t is in s. Find the wave's amplitude, frequency, wavelength, speed, and direction of motion.

★ **53** A wave of wavelength 40.0 cm and amplitude 4.00 cm propagates along a string at 10.0 m/s, in the positive x direction. At $t = 0$, the displacement at the origin is zero and the string is moving upward. Find the displacement at $x = 3.00$ m at $t = 0.500$ s.

★★ **54** The wave form shown in Fig. 16-47 travels along the positive x-axis at 10.0 m/s. Find the displacement y at $x = 25.0$ m at $t = 20.0$ s.

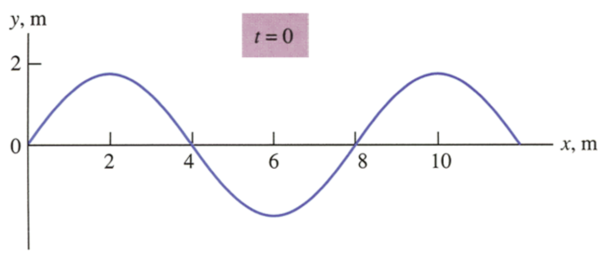


Fig. 16-47

16-6 Superposition of Waves; Beats; Standing Waves

55 Find an expression for the displacement y produced when there are two waves present with displacements $y_1 = 4 \sin(2t - 5x)$ and $y_2 = 2 \sin(2t - 5x)$.

56 Two upward wave pulses are generated at opposite ends of a string and travel toward each other. One has an amplitude of 5.00 cm and the other has an amplitude of 3.00 cm. What is the maximum displacement of any particle of the string, and when does this displacement occur?

57 A positive (upward) wave pulse and a negative (downward) wave pulse are generated simultaneously at opposite ends of a string. The maximum displacements of the positive and negative pulses are 5.00 cm and 3.00 cm respectively. What is the maximum displacement at the midpoint of the string?

58 Two small loudspeakers are connected to a single source—an 860 Hz sine-wave generator—so that they are coherent. Suppose that initially the speakers are side by side, a few centimeters apart, equidistant from a listener 2.00 m in front of the speakers.

(a) One of the speakers is slowly moved straight back away from the listener until at some point she hears almost no sound. How far is the speaker moved?

(b) How much farther back should the speaker be moved for the sound to be about as loud as it was initially?

★ **59** Two coherent harmonic sound sources produce destructive interference at a point that is 1.23 m from one source and 1.26 m from the other. What are the three lowest possible frequencies of the waves?

60 Find the beat frequency produced by an 800 Hz source and an 804 Hz source.

61 A piano tuner simultaneously strikes a 262 Hz tuning fork and the middle C key on a piano and hears beats with a beat period of 0.500 s. What are the possible frequencies of the slightly out-of-tune key? Should the tension in the piano wire be changed in such a way as to shorten or to lengthen the period of the beats?

62 Find the wavelengths of the three longest standing waves that can be formed on a 1.00 m long string fixed at both ends.

63 Find the first three harmonics of a 80.0 cm guitar string if the speed of waves on the string is 704 m/s.

★ **64** Use the trigonometric identity $\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \sin\beta\cos\alpha$ to prove that the superposition of two harmonic waves traveling in opposite directions produces a standing wave; i.e., prove that if

$$y_1 = A\sin\left[2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)\right]$$

and $$y_2 = A\sin\left[2\pi\left(\frac{t}{T} + \frac{x}{\lambda}\right)\right]$$

then $$y = y_1 + y_2 = 2A\sin\left(\frac{2\pi t}{T}\right)\cos\left(\frac{2\pi x}{\lambda}\right)$$

Sketch y at various points in a cycle.

65 A musician playing a violin, guitar, or other string instrument changes the fundamental frequency of one of the strings by "fingering," that is, by pressing the string against the neck of the instrument with a finger so that the length of the string is effectively shortened. To change a string's fundamental frequency from 440 Hz (A) to 512 Hz (C), how much should one shorten the string if its original length is 70 cm?

66 The E string on a guitar has a length of 66.0 cm. The string's fundamental frequency is 165 Hz. Pressing the string against one of the frets along the neck of the guitar effectively shortens the length of the string. What length will give the E string a frequency of 262 Hz (middle C)? Assume the tension is constant.

67 The longest pipe in a certain organ is 4.00 m long. What is the lowest frequency the organ will produce if the pipe is (a) open at both ends; (b) closed at one end?

68 Given that most people cannot hear sounds outside the frequency range 50 Hz to 10,000 Hz, what are reasonable minimum and maximum lengths for musical wind instruments, which are open at both ends?

★ **69** If the temperature of the air in an organ pipe drops from 300 K to 250 K, what will the fundamental frequency of the pipe be if it originally was 440 Hz?

70 At a certain point in space, two waves have displacements y_1 and y_2, which are graphed as functions of time in Fig. 16-48. Sketch the resultant displacement $y = y_1 + y_2$.

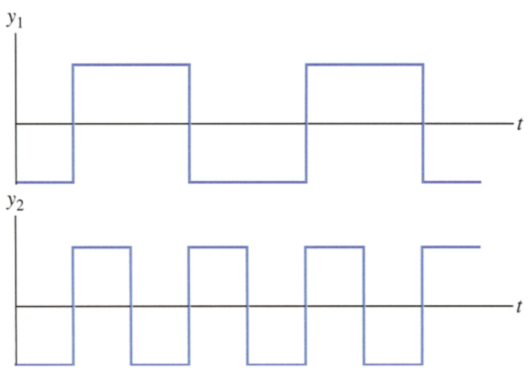


Fig. 16-48

Additional Problems

71 The Richter scale is a logarithmic scale for measuring the total energy released in an earthquake. The Richter magnitude M is defined by the equation $M = \frac{2}{3}\log\left(\frac{E}{E_0}\right)$, where E_0 is a reference energy equal to 25,000 J.

(a) How much energy was released in the 1994 Los Angeles earthquake, which registered 6.8 on the Richter scale?

(b) By what factor would the energy of this quake have to be increased in order to have a magnitude of 7.8, or of 8.8?

Fig. 16-49 Aftermath of the 1906 San Francisco earthquake.

★ **72** The 1906 San Francisco earthquake lasted 1 min and registered 8.2 on the Richter scale (defined in problem 71).

(a) What was the average power generated during this earthquake?

(b) Estimate the intensity 10 km from the center of the quake.

73 An earthquake on the opposite side of the earth from you generates waves. How long do you wait for the arrival of P waves that travel through the earth at a speed of 5.00 km/s?

★★ **74** In a harmonic water wave, the water moves along a circular path of radius A.

(a) Prove that the speed of the water v_w is related to the wave speed v by the equation $v_w = \frac{2\pi A}{\lambda} v$.

(b) By applying Bernoulli's equation in the rest frame of the wave, show that the wave speed is $v = \sqrt{\frac{g\lambda}{2\pi}}$.

75 An autofocus camera sends out an ultrasonic pulse that is reflected by an object and then detected by the camera's rangefinder. If the time delay between emission and detection is 0.0100 s, how far is the object from the camera?

★★ **76** Show that the acceleration of the water at the peak of a harmonic water wave equals g when the amplitude of the wave equals $\frac{\lambda}{2\pi}$. Would you expect the amplitude of a water wave to ever exceed this value?

★ **77** Find the maximum speed and acceleration of a particle of a string that transmits a wave of amplitude 3.00 cm and frequency 20.0 Hz.

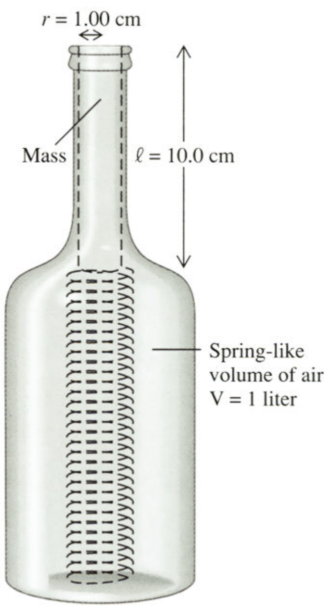


Fig. 16-50

★ **78** You can easily produce a standing wave by blowing across the top of an empty bottle. The fundamental frequency of the standing wave is surprisingly low, however. The air in the neck of the bottle oscillates in SHM as the air lower down in the bottle is alternately compressed and expanded. The air in the neck behaves like a mass attached to a spring. The springlike force is provided by the lower air. The air in the neck must move a considerable distance before there is a significant opposing force from the compressed lower air. Thus the force constant is small, and the resonant frequency $\frac{1}{2\pi}\sqrt{\frac{k}{m}}$ is low. A detailed mechanical analysis yields the following approximate formula for the resonant frequency: $f = \frac{1}{2}\left(\frac{vr}{\sqrt{\pi \ell V}}\right)$, where v is the speed of sound and r, ℓ, and V are as indicated in Fig. 16-50.

(a) Find the fundamental frequency of the bottle shown.

(b) Find the minimum length of an organ pipe having the same fundamental frequency.

★ **79** We perceive the direction of sound sources through slight time differences between the arrival of waves at each ear. The waves reach the ears simultaneously when the sound source is directly in front of the listener.

(a) Find the time difference when sound comes from a distant source at an angle of 30.0° relative to the forward direction, as indicated in Fig. 16-51.

(b) What would the time difference be if the source were directly behind the listener? How can the listener distinguish between sources directly in front and those directly behind? Will the effect of rotating the head a few degrees give the same result for sources in front and those behind?

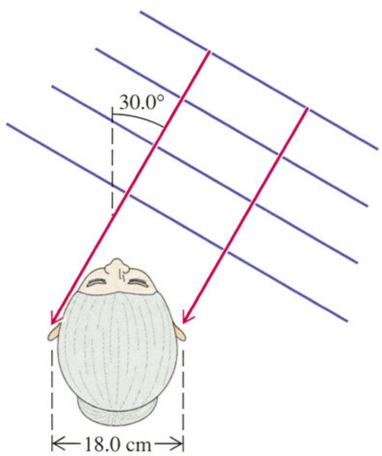


Fig. 16-51